A simple and efficient way to store many messages using neural cliques

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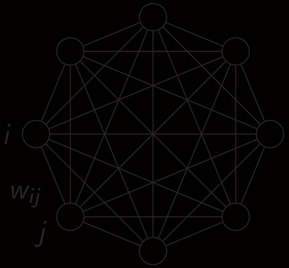
Hopfield neural networks

Associative memories: principle

- Learn M messages,
- Retrieve a learnt message in presence of erasures or errors.

The Hopfield network



Example with $n = 8$ neurons.

- Learning (static): M binary ($\{-1; 1\}$) messages $\mathbf{d}^m$: $w_{ij} = \sum_{m=1; i,j \leq n}^{M} d_i^m d_j^m$ (w_{ij} is specified on $M+1$ levels),
- Retrieving (iterative): repeat $\forall i, v_i \leftarrow \mathrm{sgn}(\sum_{j \neq i} v_j w_{ij})$.

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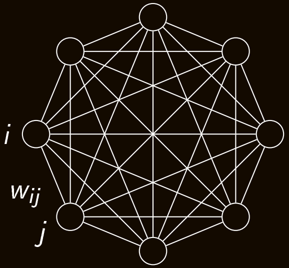

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Bounds and limitations

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- Diversity (number of learnt messages): $\frac{n}{2log(n)}$ sublinear,
- Capacity: $\frac{n^2}{2log(n)}$ subquadratic,
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- $\Rightarrow$ Efficiency $\frac{\text{capacity}}{\text{binary information used}} \approx \frac{1}{log(n)log_2(M+1)} \xrightarrow[n\to\infty]{} 0$.

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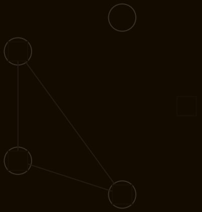
Why cliques?

Strong separability

- Cliques are fully interconnected subsets of nodes,
- Two distinct cliques with c nodes have at least $2(c-1)$ different connections:



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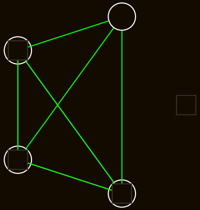

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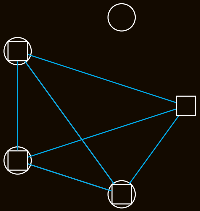

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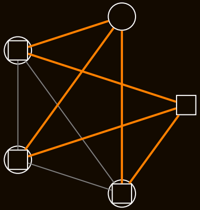

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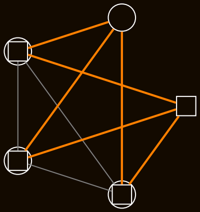

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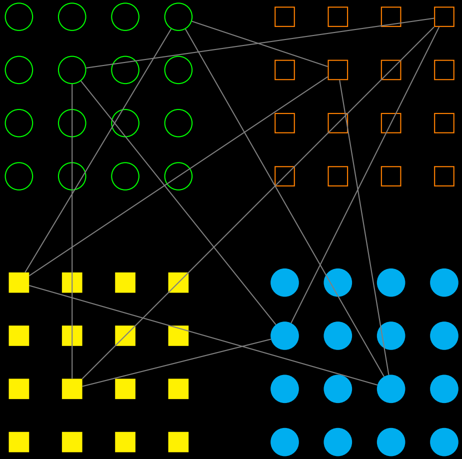

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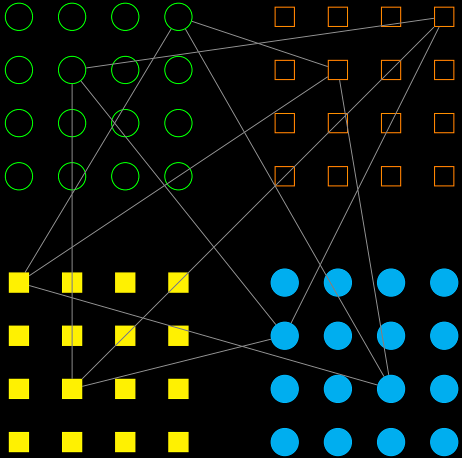

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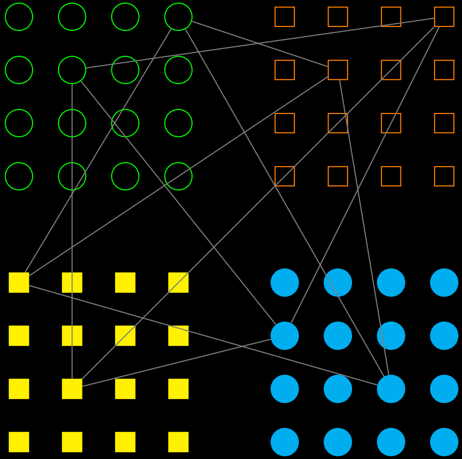

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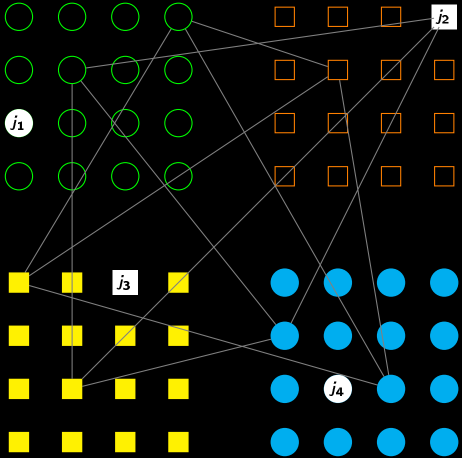

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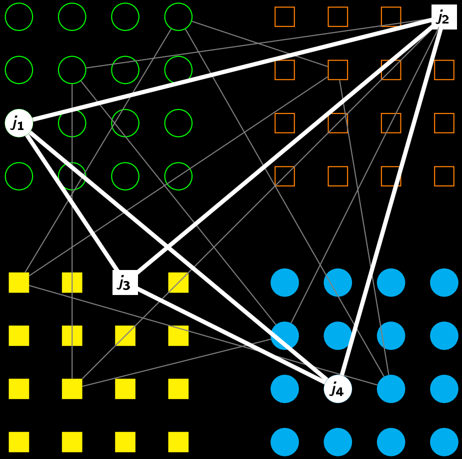

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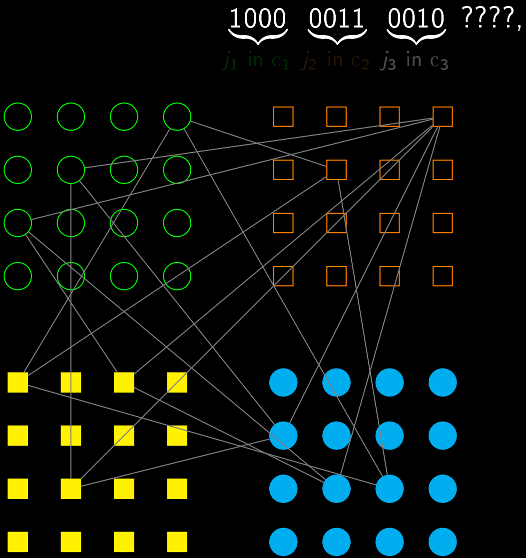

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- Possibly iterates the two last steps.

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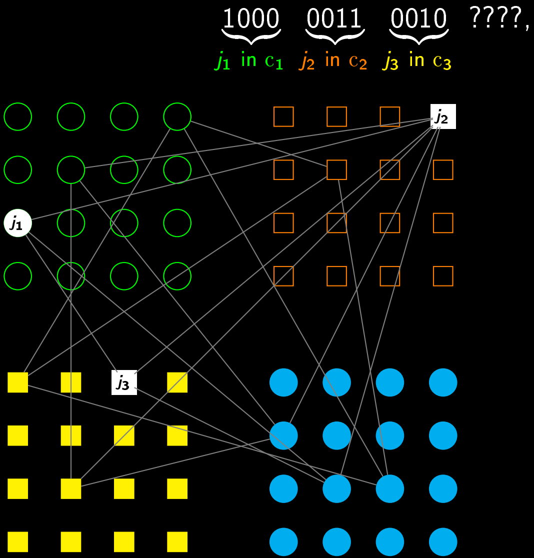

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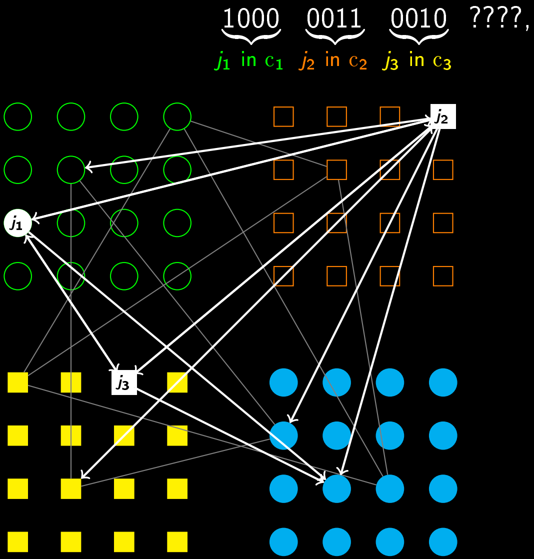

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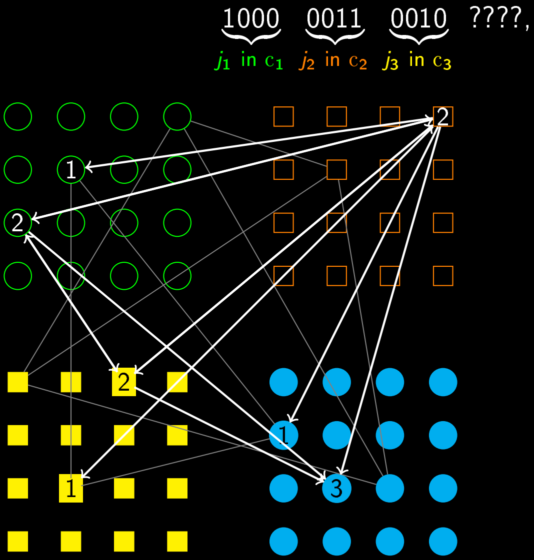

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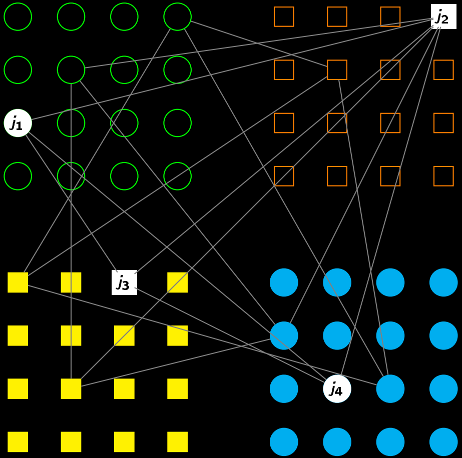

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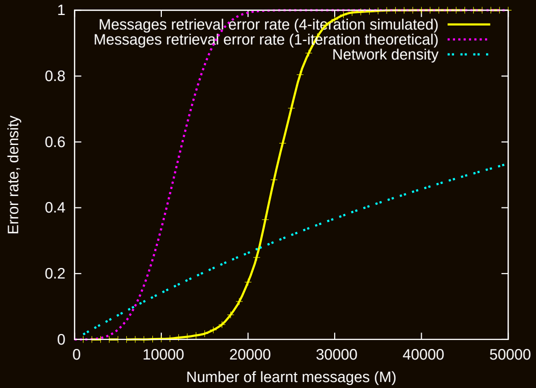
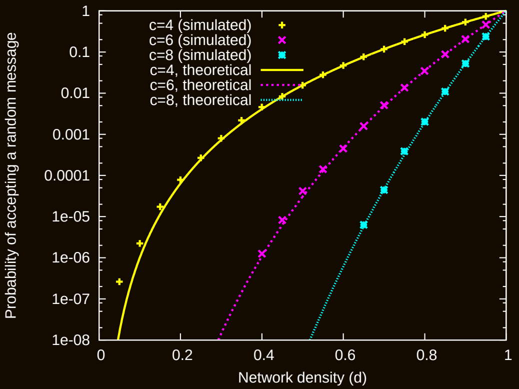


$c = 8$ clusters of $l = 256$ neurons,
Error probability while retrieving learnt messages with 4 out of 8 clusters with no provided information.

- Gain in comparison with the Hopfield network: 250 in diversity, 20 in capacity, and 20 in efficiency ($2.6\% \rightarrow 52\%$).

Performance

Classification

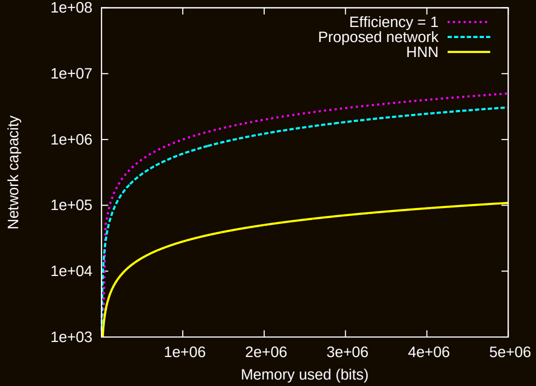


Various c values and $l = 512$ neurons per cluster, Error probability of second kind (probability to accept a non learnt message).

- Dramatic increase in performance compared to Hopfield network: 1071 in diversity, 52 in capacity, 52 in efficiency (2.6% $\rightarrow$ 137%),

Comparison in capacity with Hopfield neural networks

Capacity



Capacity of Hopfield neural networks and of our model (in the case of associative memory and a single iteration) in function of the amount of used information.

Conclusion

Results

- A novel neural architecture has been proposed which improves considerably diversity, capacity and efficiency,
- Biological plausibility has also been increased: binary connections, cliques over clusters, few iterations...

Further details

- See V. Gripon and C. Berrou, "Sparse neural networks with large learning diversity", to appear in IEEE Trans. on Neural Networks.
- Available online at http://arxiv.org/abs/1102.4240 .

Openings

This new kind of recurrent neural networks offer many openings in computational intelligence.

Thank you for your attention!