Random clique codes

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Michael Rabbat

McGill University

August 28th, 2012

Co-authors

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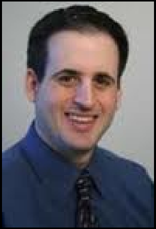
McGill University



Warren J. Gross
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Motivation

Code based associative memories

Do not miss Claude Berrou's presentation Wednesday 2:10pm.

Associative memories and erasure correcting decoders

In both cases, retrieve missing pieces of information.

Neural networks as efficient decoders

Associative memories can be efficiently implemented using neural networks.

Motivation

Build asymptotically good (non-zero minimum distance, non-zero rate) non-linear codes with efficient decoders and polynomial representations that are suitable for use with neural networks.

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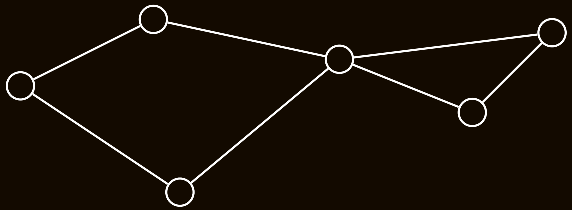
What is a clique?

Definition

Subset of vertices in a graph that are fully inter-connected.

Example



Idea

Use cliques to represent codewords.

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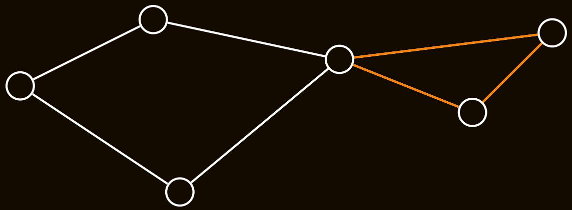

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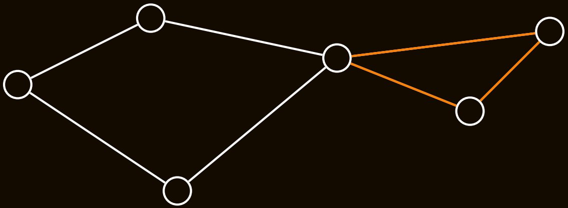

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From codes to cliques

Example of 4-ary code

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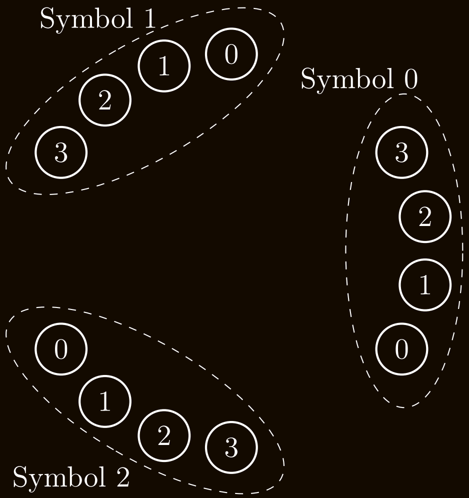
111
123
203
232
312
320

$\xrightarrow{\psi}$

Associated cliques



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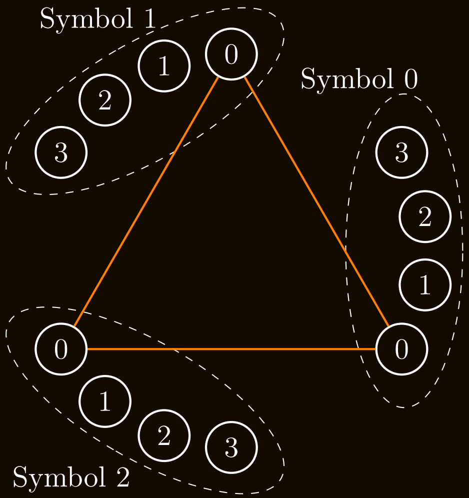

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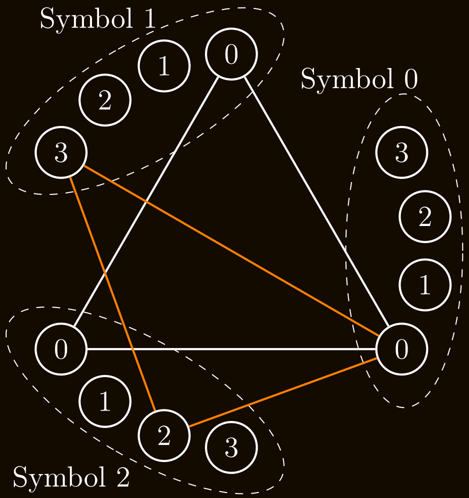

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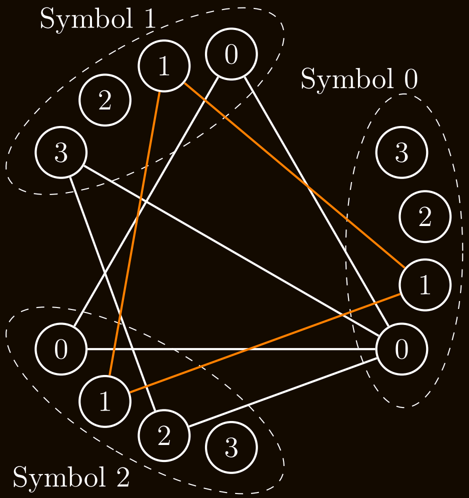

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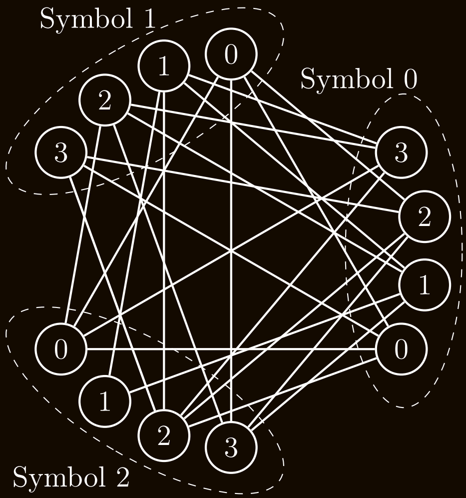

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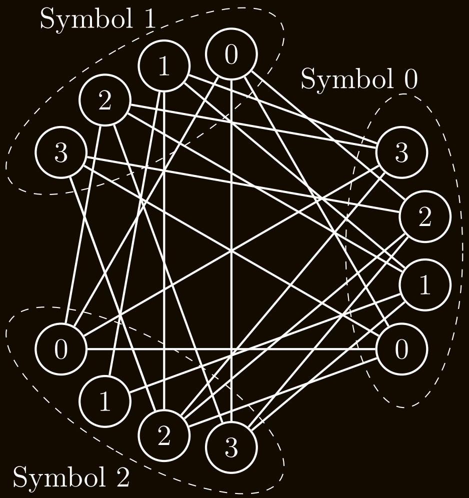
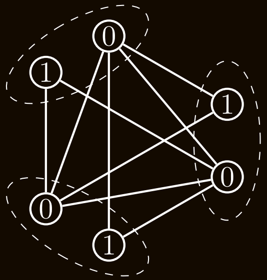


Clique codes

From cliques to codes



$\downarrow \psi^{-1}$

$$\mathcal{C}_1 = \left\{ \begin{array}{c} 000,010, \\ 001,100 \end{array} \right\}$$

$$\mathcal{C}_2 = \left\{ \begin{array}{c} 010, \\ 001,100 \end{array} \right\}$$

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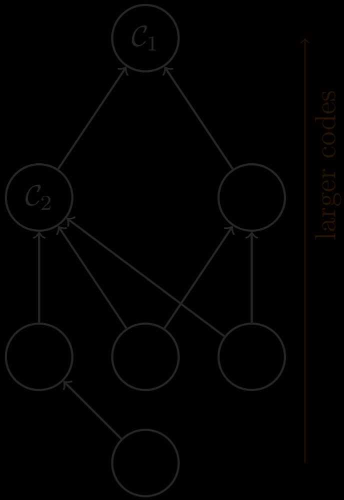
Complete Partial Order



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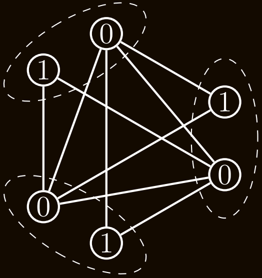

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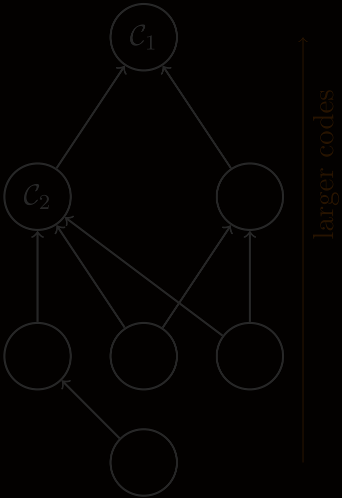

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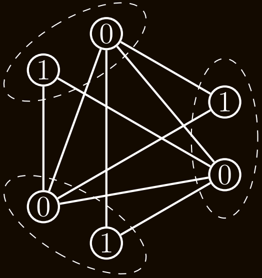

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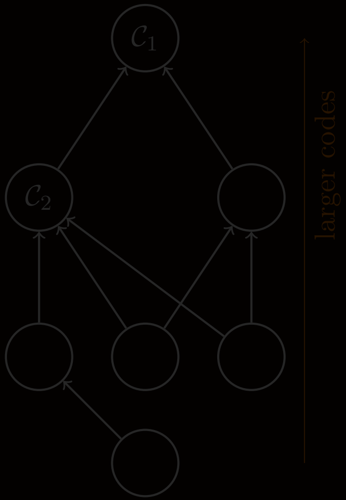

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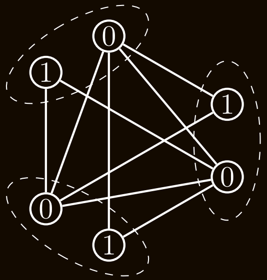

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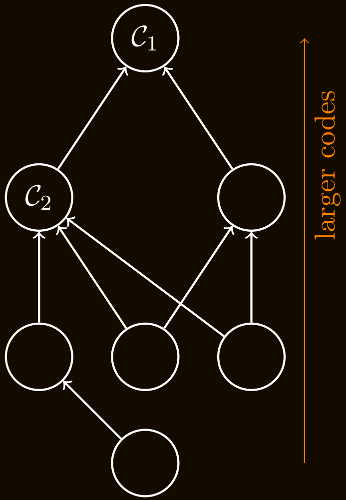

Random clique codes

Definition

Random clique code $\triangleq$ Maximum clique code associated with an Erdős–Rényi random graph

Erdős–Rényi graph

Each edge exists independently with probability p.

Number of codewords

Parameters

	In the graph	In the codes
n	Number of clusters	Length of codewords
l	Vertices per cluster	Alphabet size
p	Density of edges	$---$

Distance between cliques ⇔ Hamming distance between codewords.

Number of codewords/cliques

$$E(\#\mathcal{C}) = p^{\binom{n}{2}} \cdot \ell^n$$ Our choice: p fixed, $(n, l) = f(p)$

Remarks

- For a fixed p, an exponential number of codewords. . .
- A polynomial number of connections in the graph ($p\binom{n}{2}l^2$).
- The codes are non-linear.

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Minimum distance

Theorem

For p, $0 < p < 1$, fixed, $\exists \mathcal{C}$ with relative minimum Hamming distance at least $\alpha = \frac{d_{min}}{n}$ and rate R ($R \triangleq \frac{\log_\ell(|\mathcal{C}|)}{n}$) such that

$$R \geq \frac{\alpha(1-\alpha)}{1+2\alpha-\alpha^2} - \epsilon \,,$$

for any given small $\epsilon > 0$.

Sketch of the proof

- Look at the probability to have two codewords at distance $\leq d_0$.
- To avoid dependence issues, look at large family of large random clique codes,
- Prove the result to be true in average over such a family,
- Use convexity arguments to extend the result to a particular code.

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Neural networks-based algorithm

Erasure channel

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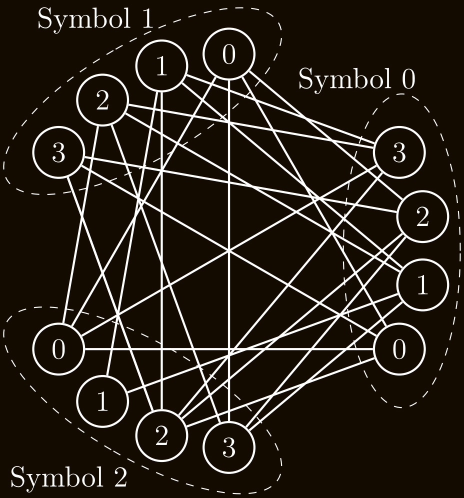
Channel

0*0

Decoder

000

Algorithm



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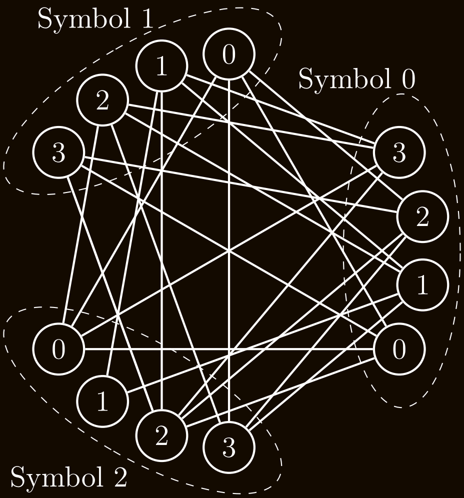

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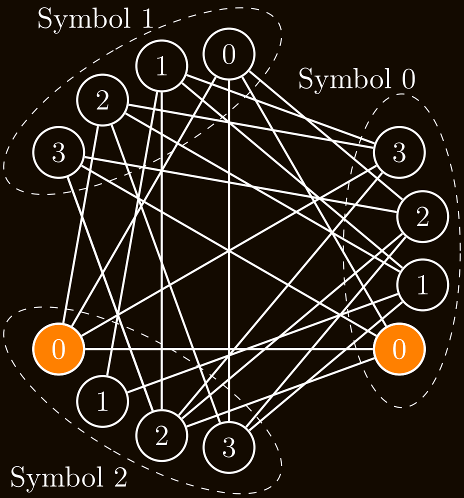

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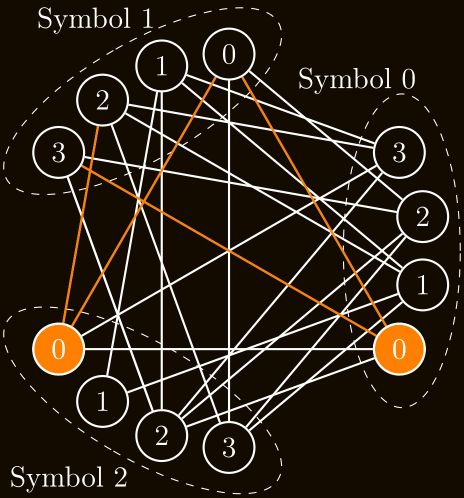

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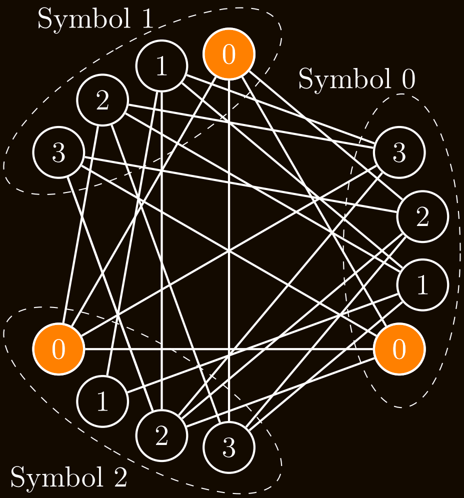

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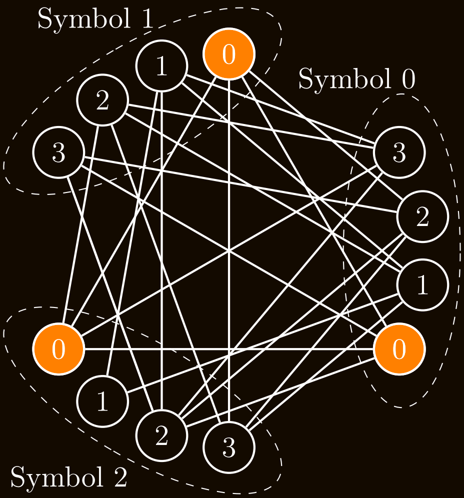
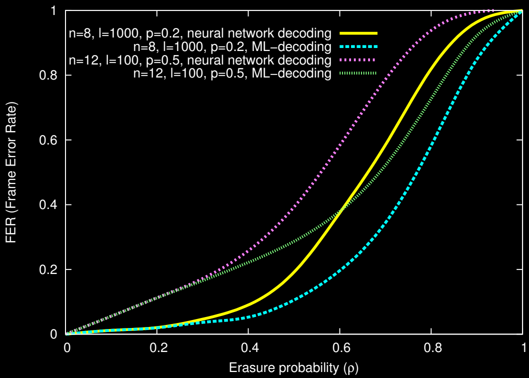


Complexities

Computational complexity	$O(n^2\ell^2)$
Memory (bits)	$\binom{n}{2}\ell^2$



Conclusions & open questions

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- Introduced a new family of random codes...
- Asymptotically good, non-linear and polynomially represented,
- They can be decoded using a small and fast neural networks based algorithm.

Open questions

- Find an efficient encoding scheme,
- Investigate other sets of parameters ($p = f(n, \ell)$),
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