

Networks of neural cliques

Vincent Gripon

Télécom Bretagne, Lab-STICC

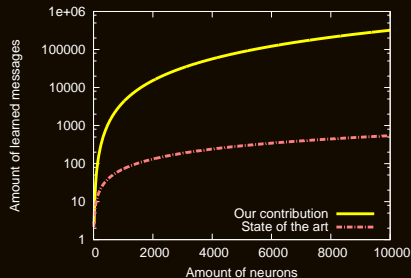
SICMA doctoral school

2011, Jul. 20th

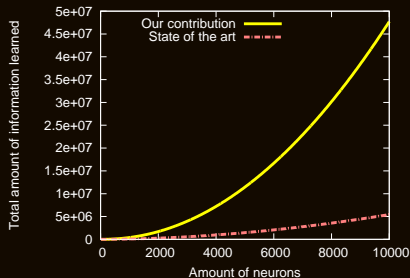
In a word...

Learning messages in recurrent neural networks

Learning diversity



Learning capacity



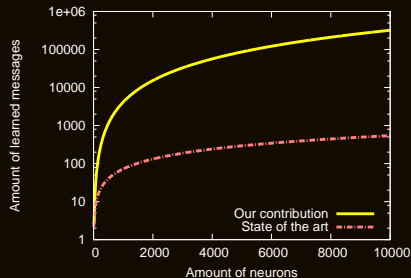
Our contribution

State of the art

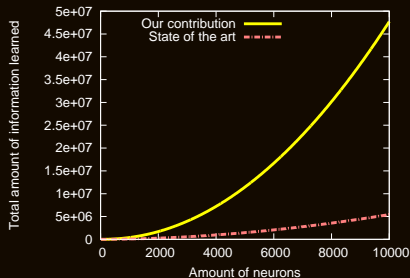
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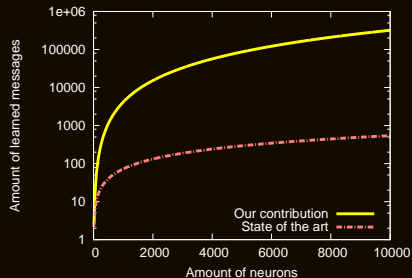
Sparsity
Error correcting code

State of the art

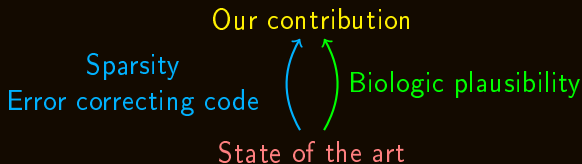
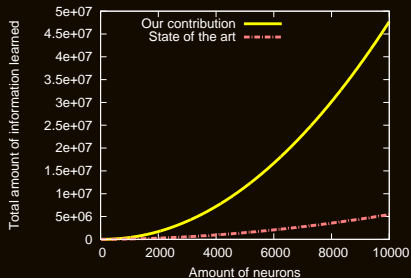
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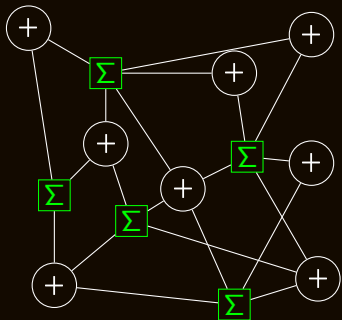


Learning capacity



Starting idea

LDPC decoder



Neocortical "decoder"



noeuds Σ = neurons
decoding = remembering
parity = ?
? = learning

1 Associative memories and error correcting codes

- Associative memory
- Error correcting codes
- Code of cliques

2 Sparse networks, principles and performance

- Learning
- Retrieving
- Performance

3 Developments

- Blurred messages
- Correlated sources
- Sparse messages
 - Global winner-take-all
 - Time synchronization

4 Conclusion, openings

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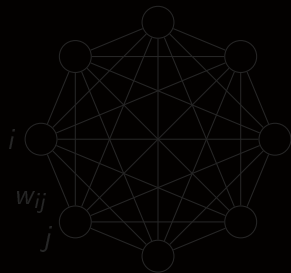
Associative memories and the Hopfield network

Associative memories

Two operations:

- **Learning** messages,
- **Retrieving** previously learned messages from part of their content.

State of the art: the Hopfield network



- Learning: M binary messages \mathbf{d}^m :

$$w_{ij} = \sum_{m=1, i \neq j}^M d_i^m d_j^m,$$

- Retrieving: iterates

$$y_i = \text{sgn} \left(\sum_{j=1}^N y_j w_{ij} \right)$$

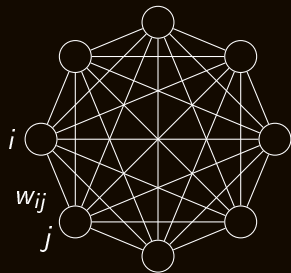
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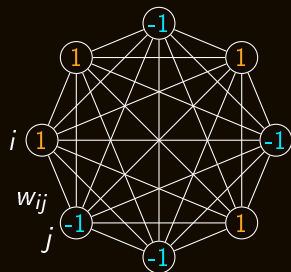
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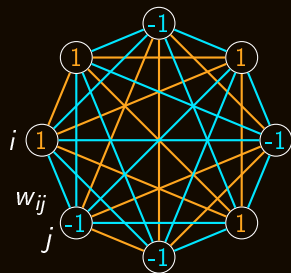
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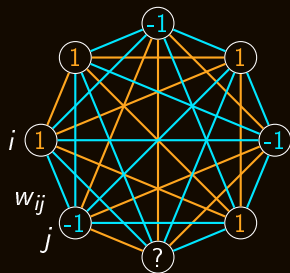
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Hopfield networks (n neurons \longleftrightarrow)

- **Diversity** : $M = \frac{n}{2\log(n)}$, \leftrightarrow
- **Capacity** : $\frac{n^2}{2\log(n)}$, $\blacksquare = \blacksquare$
- Total amount of required memory: $\binom{n}{2} \log_2(M+1)$, $\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$
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- Sensitive connections, length of messages = size of the network, messages and their inverse are learned at the same time...

Example with $n = 790$: 

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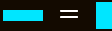


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Example: the thrifty code

- Code containing only binary words with a single "1"



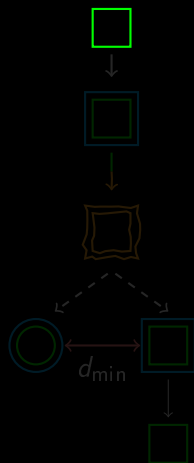
- Drawback: $d_{\min} = 2$



- But how to decode and minimize the number of errors?



- The code can be decoded by the standard algorithm

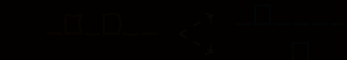


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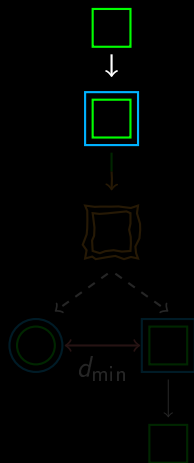
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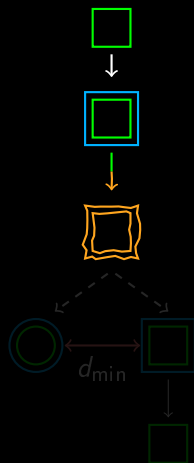


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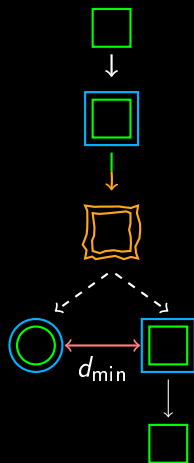
- Code containing only binary words with a single "1"
- Drawback: $d_{\min} = 2$
- But can be decoded and minimum distance can be increased by adding more words
- The code can be extended by the dual code



Error correcting codes

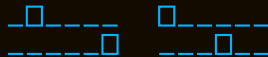
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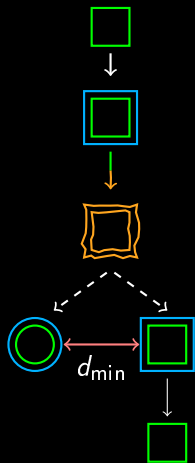
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- But **easy to decode** and **minimise the energy**:



- These codes can be associated like the distributed codes...



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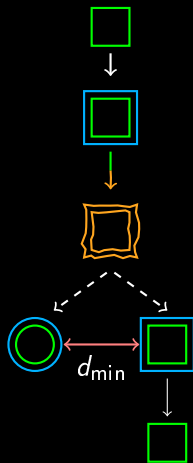
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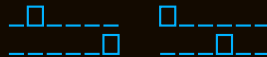


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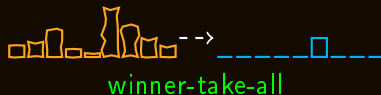
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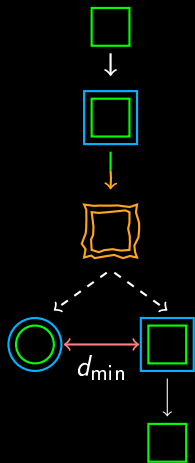
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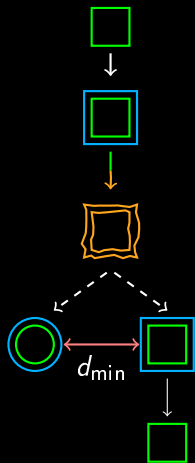
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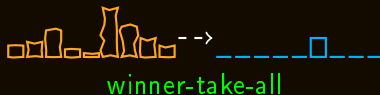
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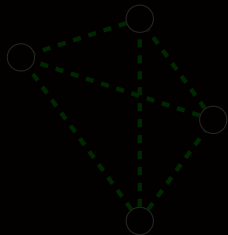
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Codes made of cliques of constant size

Example: codewords = 4 nodes cliques

Clique

Set of nodes that are all connected one to another.



2 distinct nodes
 $\Rightarrow d_{\min} = 2$ edges

Codes of cliques of size $c \ll n$

$$n \cdot d_{\min} = 2(c-1) \approx 2c$$

$$n \approx F = rd_{\min} \approx 2r$$

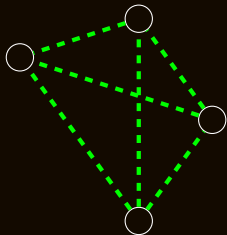
→ Cliques are codewords of a very interesting error-correcting code

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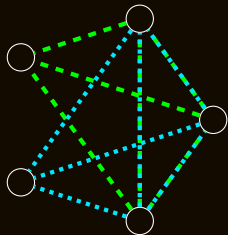
\Rightarrow number of nodes is bounded by the number of edges in a clique

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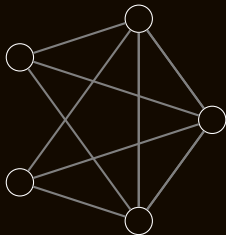
$$n = 2c \Rightarrow \frac{2c}{2c} = \frac{2(c-1) \cdot 2c}{2c} = 2(c-1)$$

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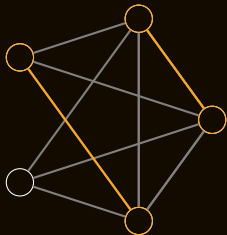
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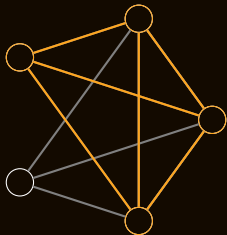
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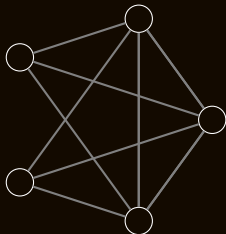
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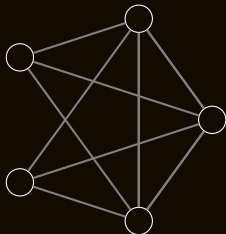
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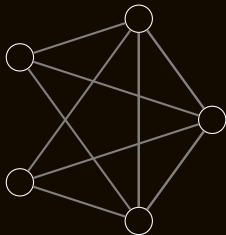
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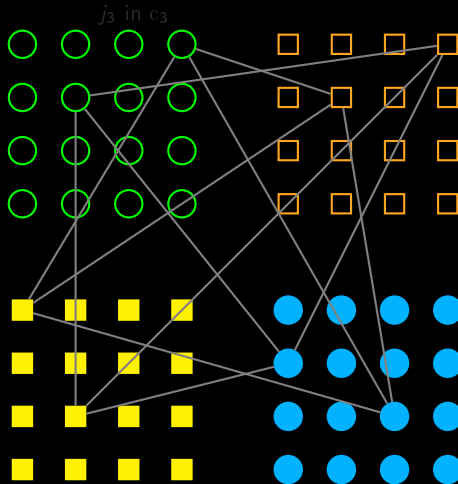
3 Developments

- Blurred messages
- Correlated sources
- Sparse messages
 - Global winner-take-all
 - Time synchronization

4 Conclusion, openings

Our model: learning

- Example: $c = 4$ clusters made of $l = 16$ neurons each,
- $\underbrace{1000}_{= 8} \underbrace{0011}_{= 3} \underbrace{0010}_{= 2} \underbrace{1001}_{= 9}$,



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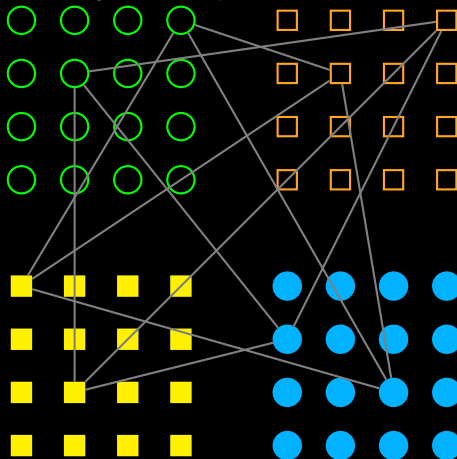
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$j_2 \text{ in } c_2$

$j_3 \text{ in } c_3$

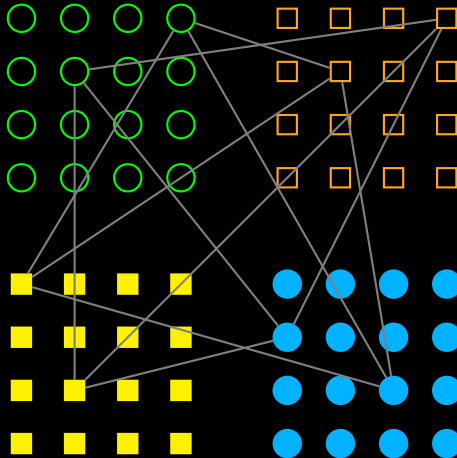
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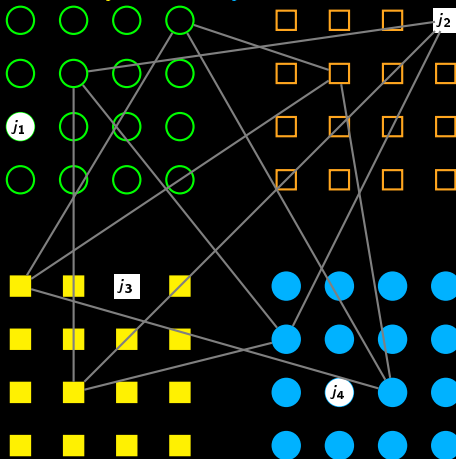
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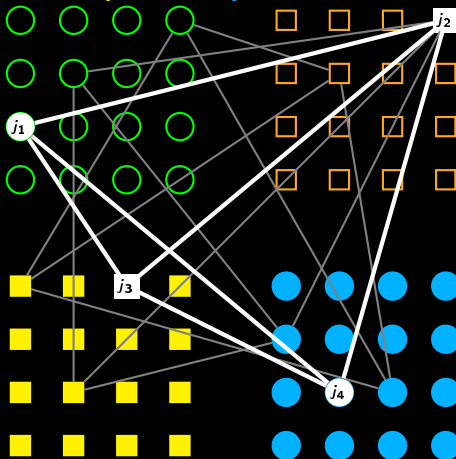
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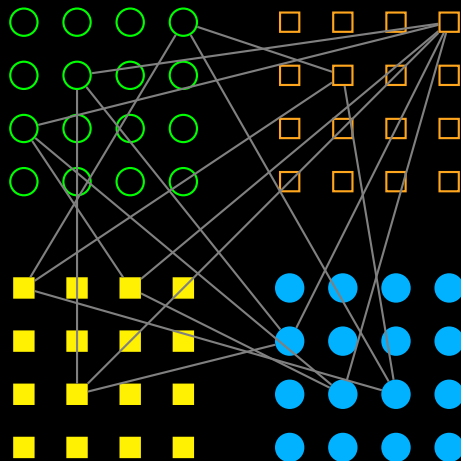
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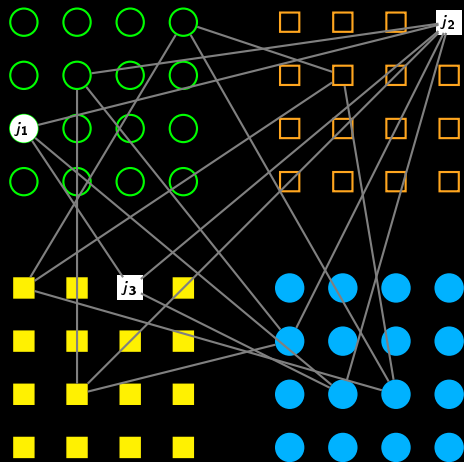
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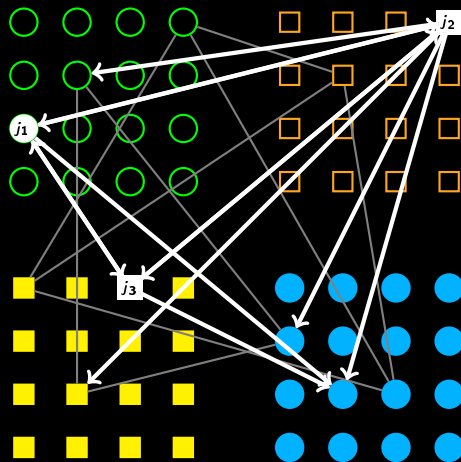
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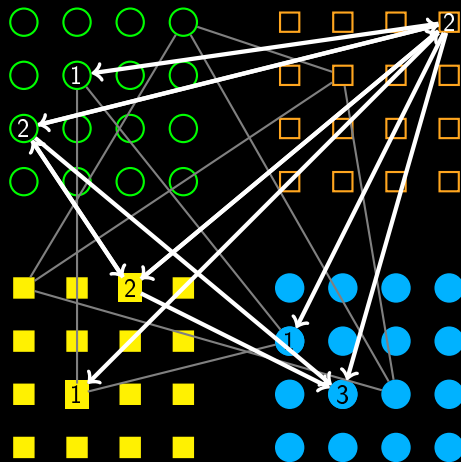
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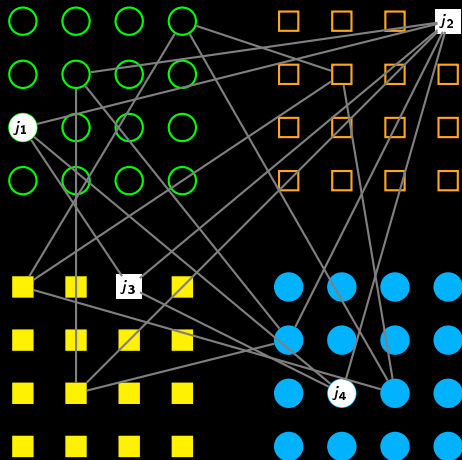
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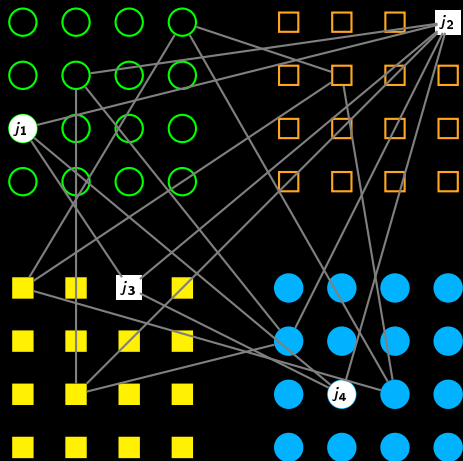
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Introducing a new parameter

- Density d is the ratio of the number of used connections to the total number of possible ones,
- If messages are i.i.d.: $d \approx 1 - \left(1 - \frac{1}{I^2}\right)^M$.

Curves

Remarks

- $d = 1$: no more distinction between learned and not learned messages,
- $d = f(I, M)$, not depending on c ,
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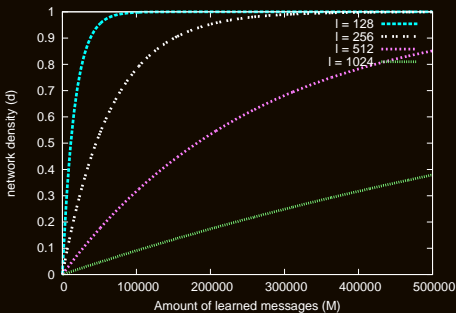
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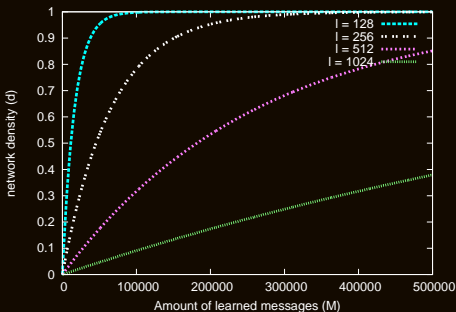
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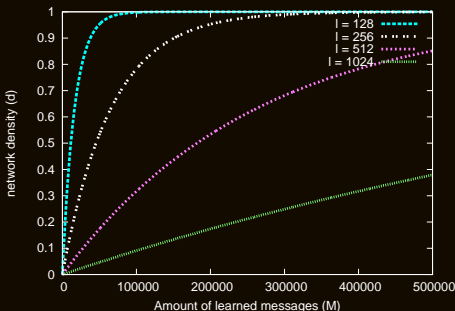
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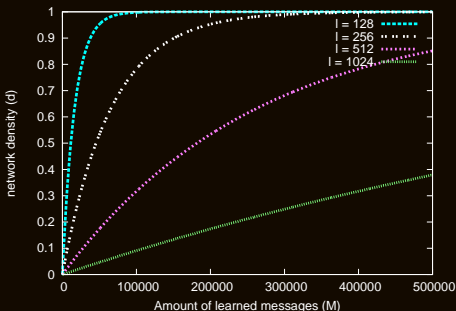
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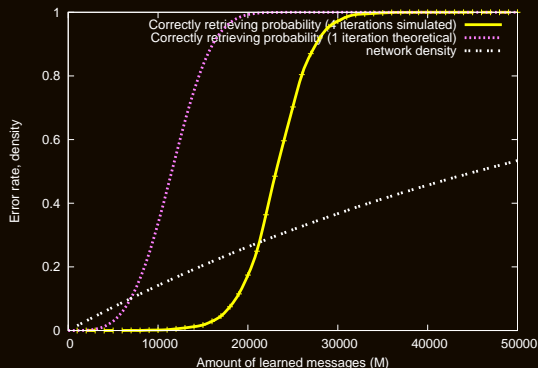


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Performance (1/3)

As an associative memory



$c = 8$ clusters of $l = 256$ neurons each (\sim messages of 64 bits),
Error probability when retrieving messages half erased.

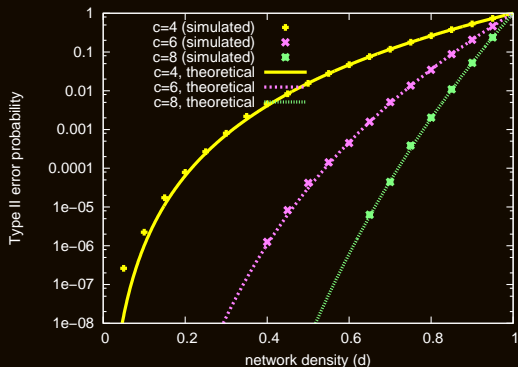
Hopfield network ($n = 790$)



Our network



Classification



Second kind error rate for various sizes of clusters c and for $l = 512$ neurons per cluster.

Hopfield network ($n = 740$)

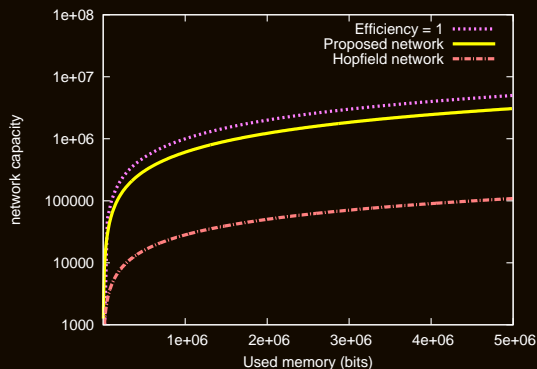


Our network



Comparison of capacities of our network and of the Hopfield one

Performance (3/3)



Comparison of the capacities of the Hopfield network with ours (as associative memories) and for the same amount of memory used.

A word about plausibility

Analogies

Our network		Neuroscience litterature
Cliques of neurons	↔	Neural cliques
Local decoding	↔	Winner-take-all
Clusters	↔	Neocortical columns
Thrifty code	↔	Specific neurons

Limitations

- Necessity to provide a perfect – yet incomplete – content
- Messages must not be correlated
- Clusters must be large and few
- Constant messages length
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1 Associative memories and error correcting codes

- Associative memory
- Error correcting codes
- Code of cliques

2 Sparse networks, principles and performance

- Learning
- Retrieving
- Performance

3 Developments

- Blurred messages
- Correlated sources
- Sparse messages
 - Global winner-take-all
 - Time synchronization

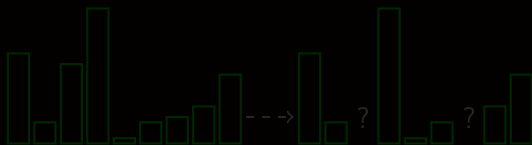
4 Conclusion, openings

Blurred messages

Limitation

Partial messages must contain perfect information.

Noise model



Soft decoding

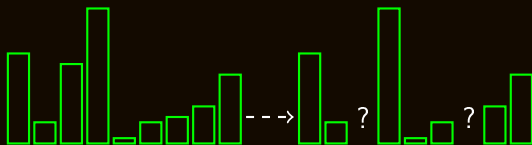


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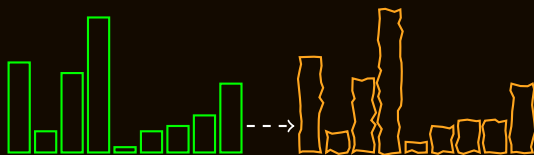


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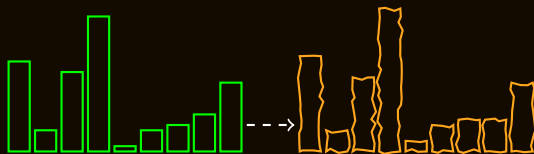


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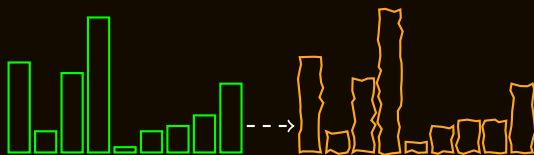


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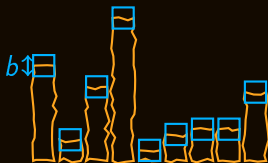
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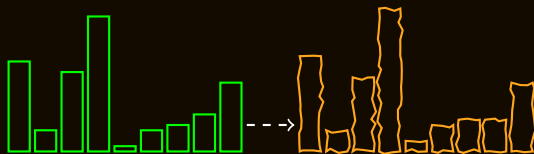


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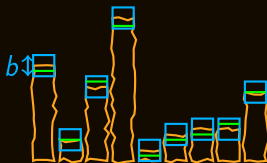
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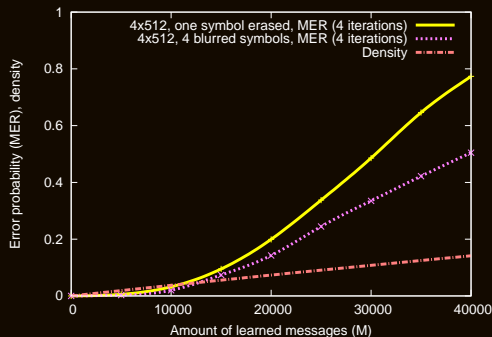
Noise model



Soft decoding



Simulations



Comparison of performance when messages are partially erased and when they are blurred ($b = 5$).

Why performance are better?

- Erasing: \searrow competitive cliques ($\approx l$) \nearrow probability ($\approx d^{c-1}$),
- Bruit : \nearrow competitive cliques ($\approx b^c$) \searrow probability ($\approx d^{\frac{c(c-1)}{2}}$).

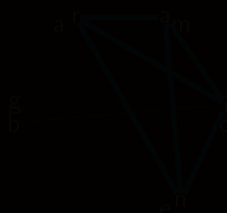
Correlated messages

Limitation

With correlations grows the number of Type II errors.

Fighting correlation by adding random redundancy

- There are two effects of correlation:
 - An inescapable effect: *brain* and *train* are learned \rightarrow *rain ?
 - Another effect coming from our network:



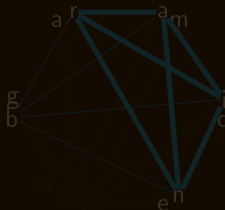
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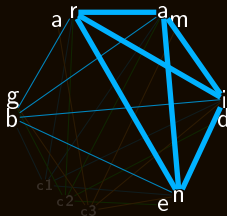
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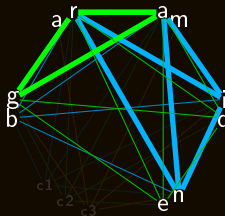
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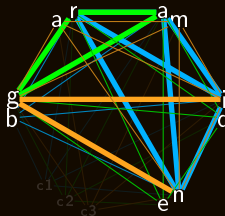
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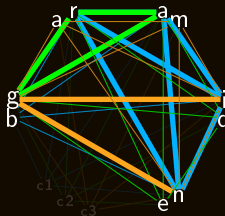
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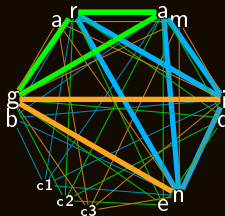
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brain +c1

grade +c2

gamin +c3

grain +c?



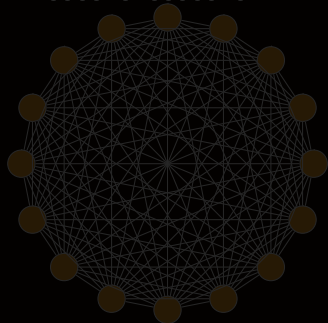
Towards a fourth level of sparsity

Limitations

- Clusters must be large and few,
- Learned messages are all of the same length.

Illustration

0000101000001011



Idea

- Shorter messages,
- Clusters and thrifty codes,
- Sparse network,
- Sparse messages.

Solutions

- Global winner-take-all
- Time synchronization.

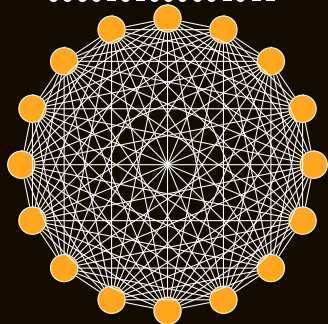
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- Shorter messages,
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- Sparse network
- Sparse network

Solutions

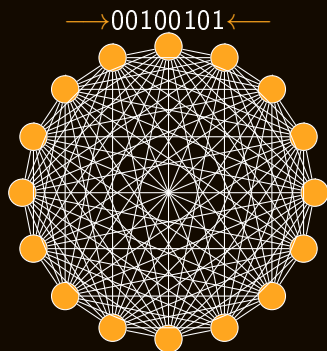
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Towards a fourth level of sparsity

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Illustration



Idea

- 1 Shorter messages,
- 2 Clusters and thrifty codes,
- 3 Sparse network,
- 4 Sparse messages.

Solutions

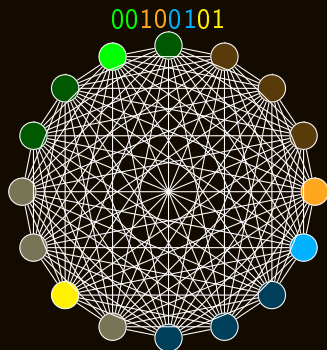
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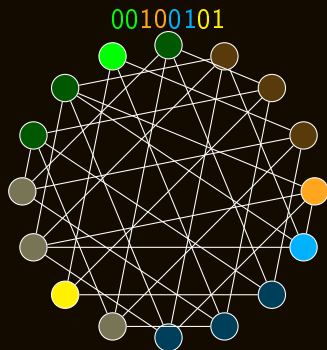
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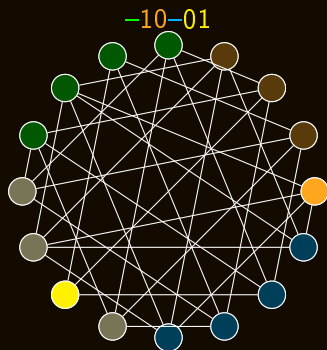
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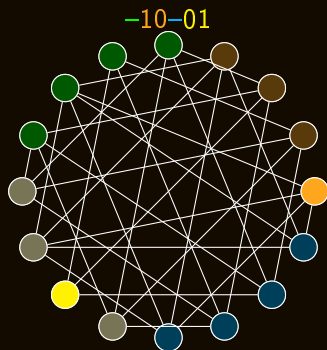
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- 4 Sparse messages.

Solutions

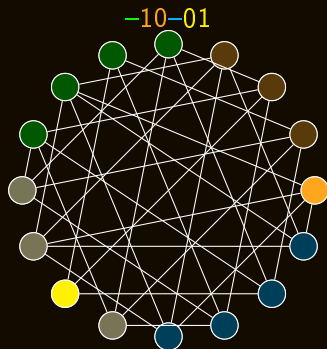
- Global winner-take-all,
- Time synchronization.

Towards a fourth level of sparsity

Limitations

- Clusters must be large and few,
- Learned messages are all of the same length.

Illustration



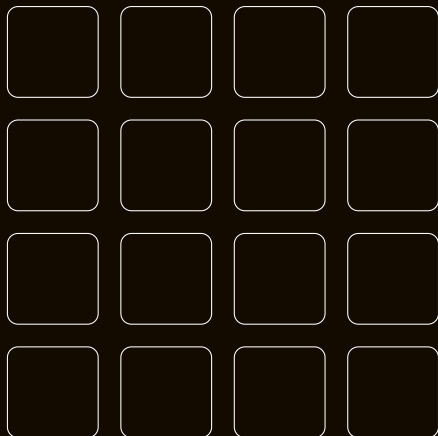
Idea

- 1 Shorter messages,
- 2 Clusters and thrifty codes,
- 3 Sparse network,
- 4 Sparse messages.

Solutions

- Global winner-take-all,
- Time synchronization.

Illustration



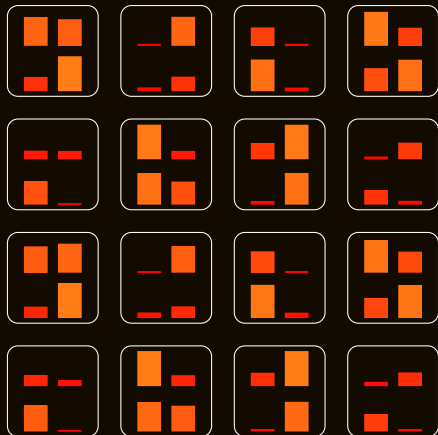
Idea

- After global message passing. . .
- After local maximum selections. . .
- Global maximum selection.

Interests

- Diversity
- Learned messages length may vary.

Illustration



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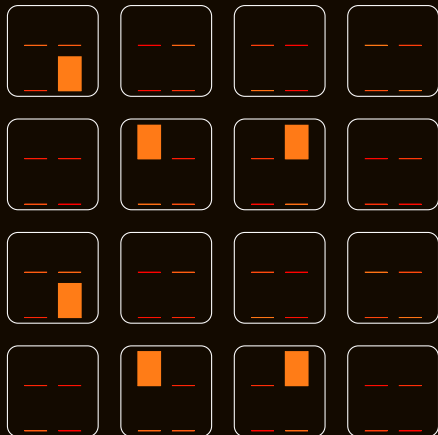
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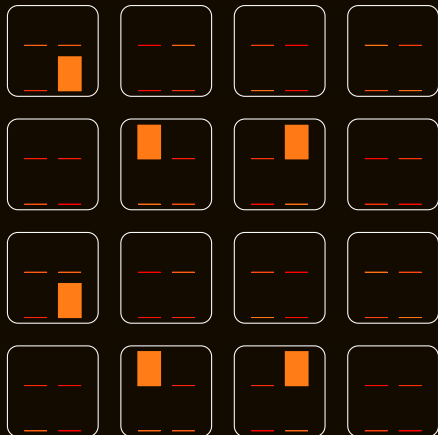
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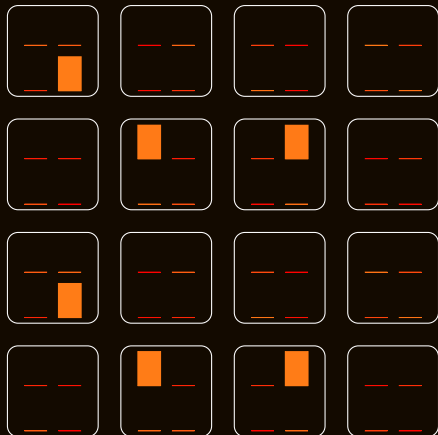
Idea

- After global message passing. . .
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Interests

- Diversity $\propto c^2$,
- Learned messages length may vary.

Illustration



Idea

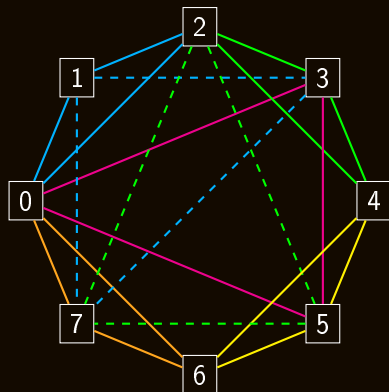
- After global message passing. . .
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Time synchronization

Illustration



Idea

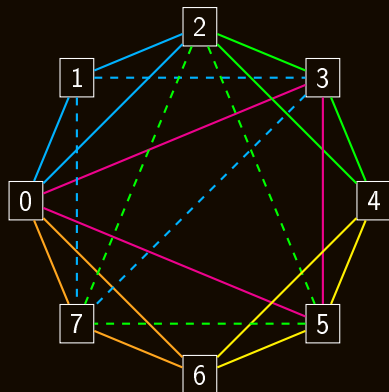
- Independent sub-networks,
- Locally: aggregation and winner-take-all,
- Globally: time coincidence detection.

Interests

Approximation possible to the modeling of diseases related to synchronization.

Time synchronization

Illustration



Idea

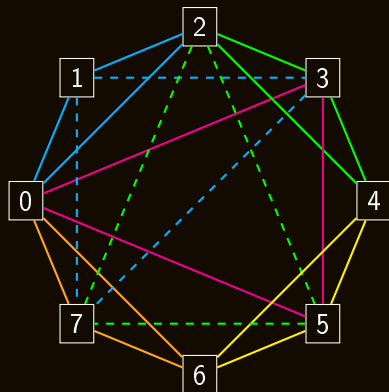
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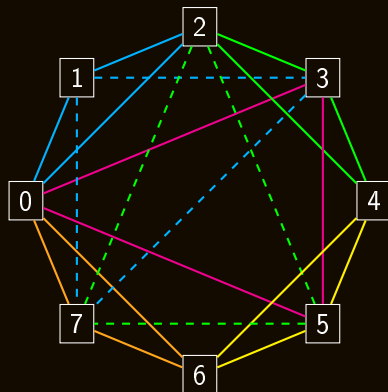
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- Independent sub-networks,
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Interests

Approximation possible to the modeling of diseases related to synchronization.

1 Associative memories and error correcting codes

- Associative memory
- Error correcting codes
- Code of cliques

2 Sparse networks, principles and performance

- Learning
- Retrieving
- Performance

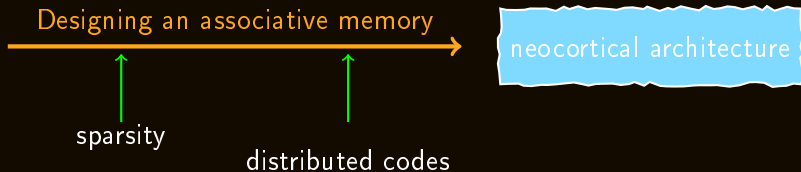
3 Developments

- Blurred messages
- Correlated sources
- Sparse messages
 - Global winner-take-all
 - Time synchronization

4 Conclusion, openings

Conclusion

Approach

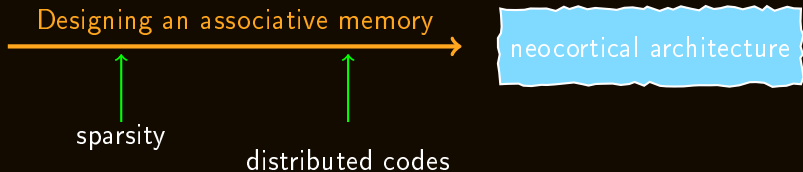


Results

- Nearly optimal capacities, substantial diversities, massively parallel architectures
- A simple, efficient, and robust architecture, with a simple learning rule
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- A trade-off required between performance and plausibility

Conclusion

Approach

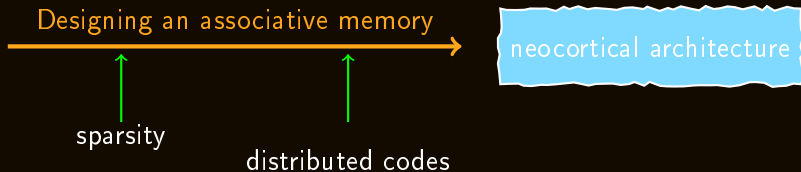


Results

- Nearly optimal capacities, substantial diversities,
- Massively parallel architecture,
- Analogies with neurobiological architecture and functioning,
- Robustness, resiliency, synchronization...
- Degrees of freedom: inhibitions, time, weights,
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Conclusion

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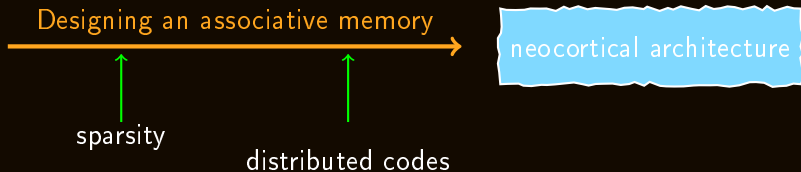


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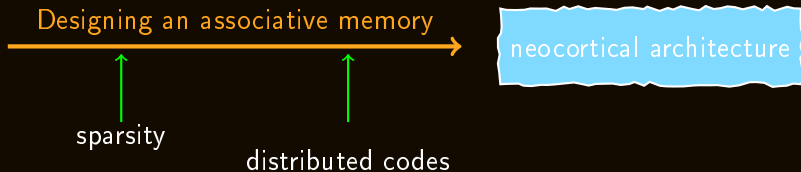


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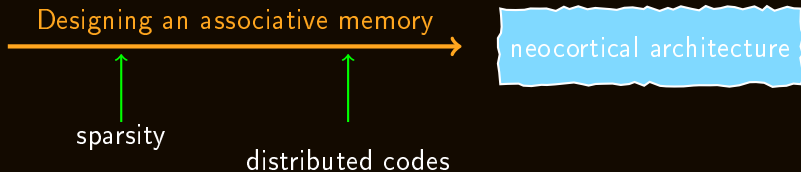


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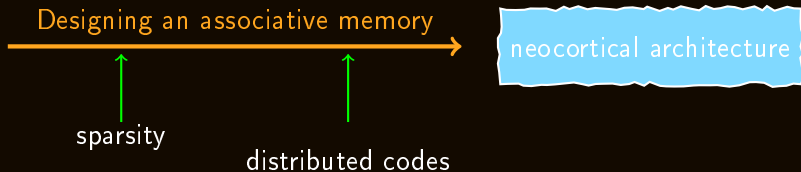


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Journal

- An article in *IEEE Transactions on neural networks*.

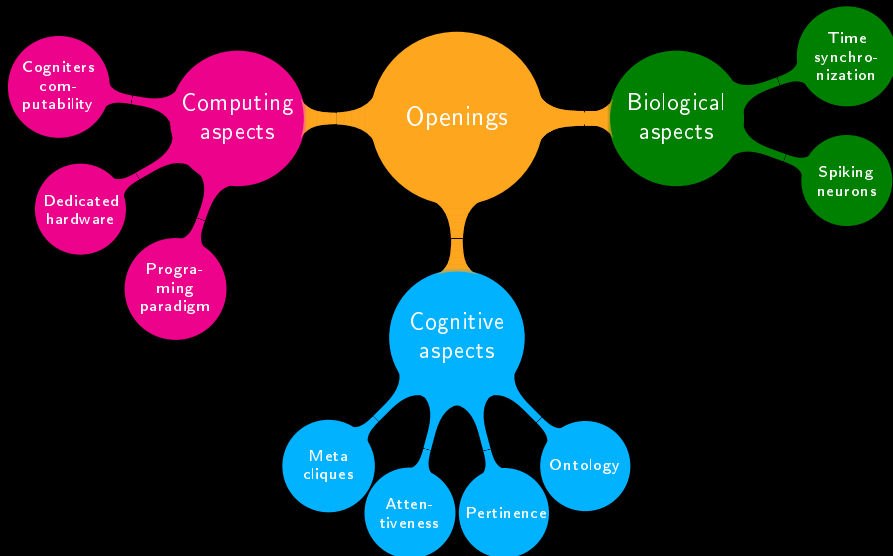
Proceedings

- A communication in *Proc. of 6th Int'l Symposium on Turbo Codes and Iterative Information Processing*,
- A communication in *Proc. of IEEE Symposium on Computational Intelligence, Cognitive Algorithms, Mind, and Brain*.

Patents

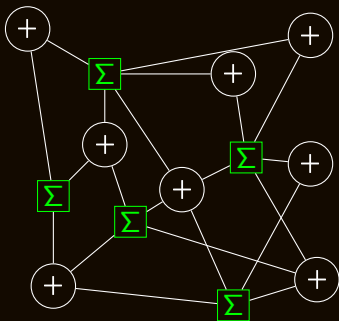
- A first patent filed in 2010: presented network,
- A second one currently being filed: learning sequences.

Openings



Thank you for your attention. I am at your disposal if you have any question.

LDPC decoder



Neocortical “decoder”

