When neural networks meet error correcting codes: towards new architectures for associative memories

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Joint work with Claude Berrou

Télécom Bretagne

April 7th, 2013

Plan

1. Biology and error correction codes
   - Starting point
   - From the neocortex to recurrent graphs : our three hypotheses
   - A few words about error correcting codes
2. A new architecture of associative memory
   - Storing
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Learning sequences
4. Current and future work
   - Storing hierarchical messages
   - Combining associative memories and classifiers
   - Towards new computation models based on information

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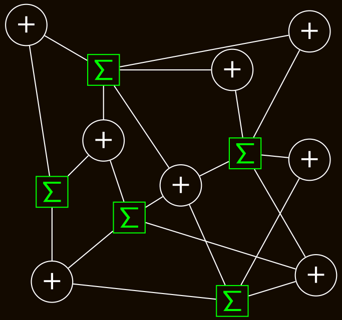
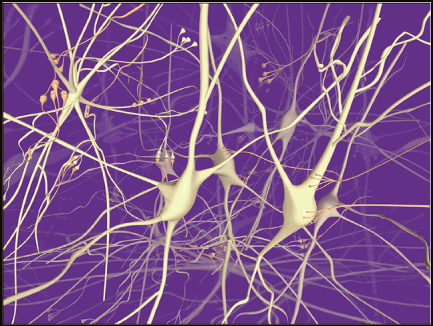
LDPC decoder

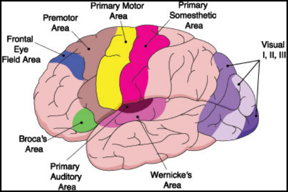


Neocortical "decoder"

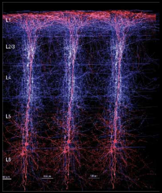


Both systems aim at retrieving a previously stored piece of information given part of its content.

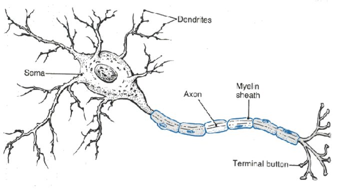
First hypothesis: the information scale



Macroscopic scale



Mesoscopic scale



Microscopic scale

Second hypothesis: redundancy

Illustration

02 29 00 12 77 12 77

02 29 00 1- 77 12 -7

Redundancy

- We lose approximately one neuron per second,
- But we remember our phone number,
- Mental information is robust,
- Therefore redundant.

Second hypothesis: redundancy

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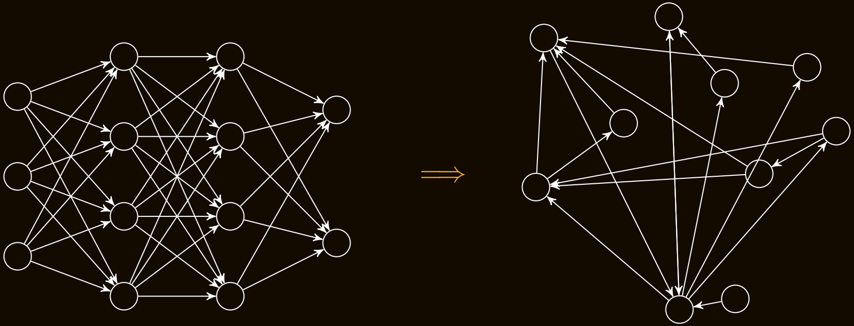
Third hypothesis: recurrent graph

The neocortex can be essentially regarded as a distributed recurrent graph.

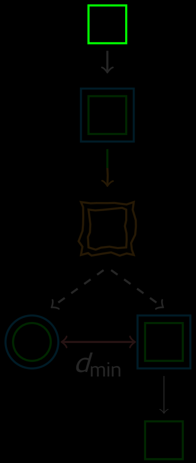
illustration



In one sentence

The neocortex is a recurrent, distributed graph of neocortical columns (fanals) that is able to store redundant pieces of information.

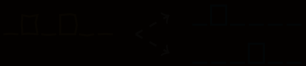
Error correcting codes



Example: the thrifty code

- Code containing only binary words with a single "1":

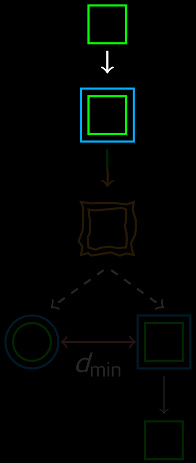
- Drawback: $d_{min} = 2$:



- But easy to decode and minimize the energy:

winner-take-all

- These codes can be associated like the distributed codes. . .



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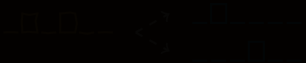

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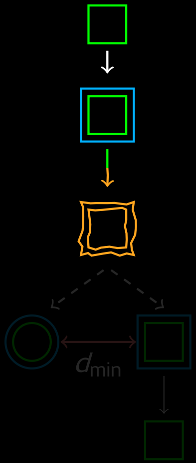

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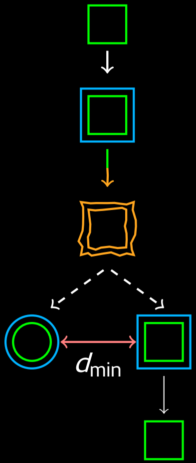

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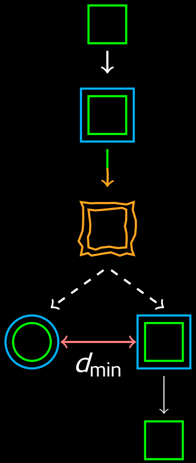

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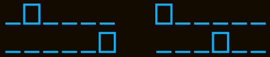

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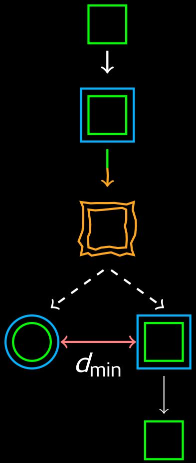

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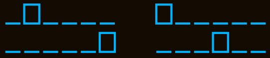

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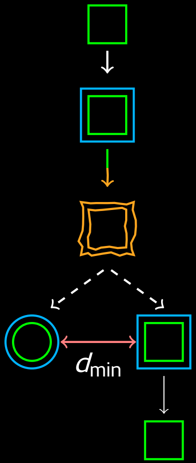

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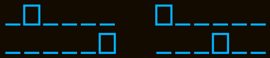

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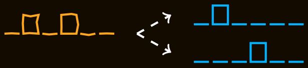

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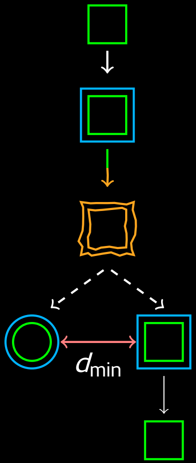

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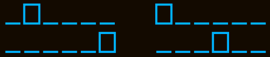

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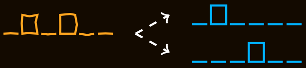

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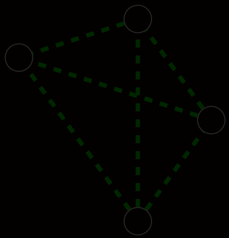
Clique

Set of nodes that are all connected one to another.

Example: codewords = 4 nodes cliques



2 distinct nodes
$\Rightarrow d_{\text{min}} = 6$ edges

Codes of cliques of size $c \ll n$

- $d_{\text{min}} = 2(c-1) \approx 2c$,
- $\Rightarrow F = rd_{\text{min}} \approx 2$,
- Cliques are codewords of a very interesting error correcting code...

Codes made of cliques of constant size

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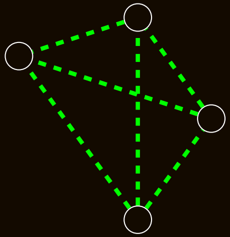

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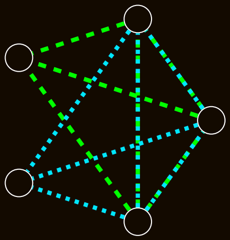

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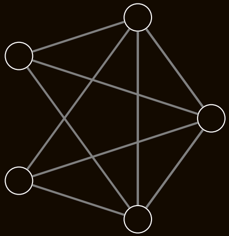

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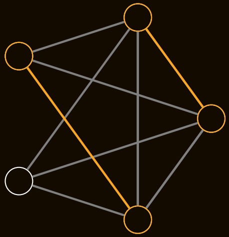

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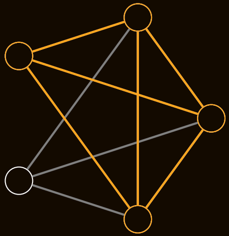

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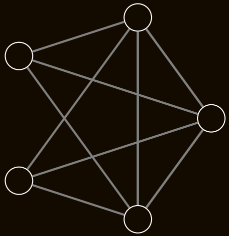

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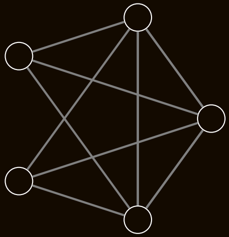

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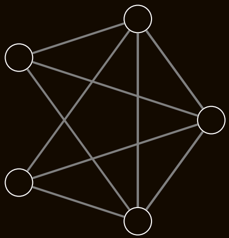

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Associative memories and the Hopfield network

What is an associative memory?

Two operations:

- Store a message,
- Retrieve a previously stored message from part of its content.

Our reference: the Hopfield network

Example:

- Store binary message -11-111-1-11
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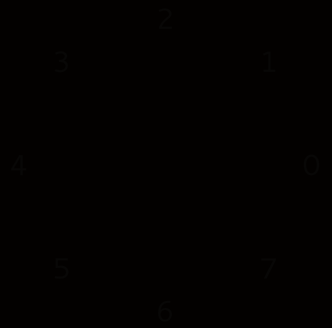

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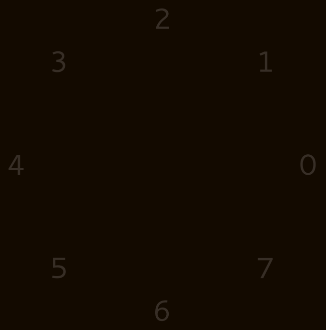

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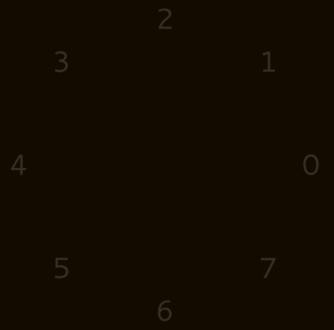

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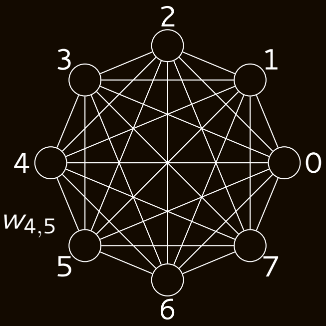

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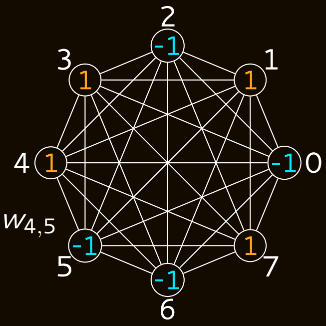

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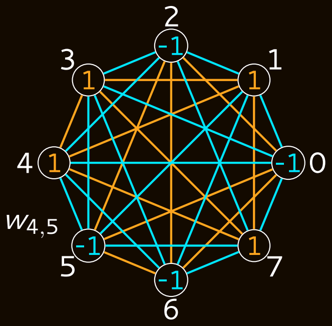

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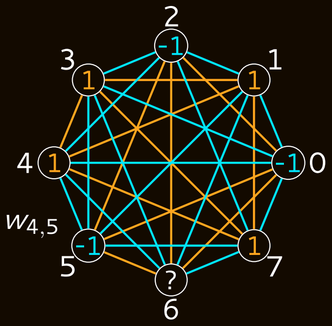

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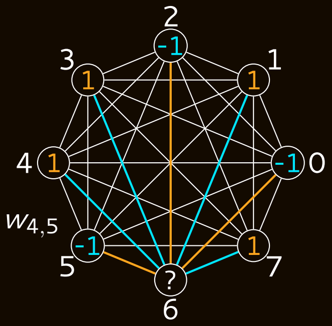

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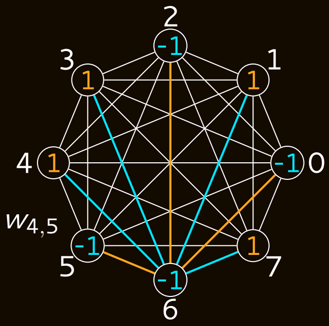

Performance and bounds

Hopfield networks (n neurons $\longleftrightarrow$)

- Diversity : $M = \frac{n}{2log(n)}$, $\leftrightarrow$
- Capacity : $\frac{n^2}{2log(n)}$,
- Efficiency $\approx \frac{1}{log(n)log_2(M+1)}$.

Example with $n = 790$: 

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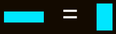
- Capacity : $\frac{n^2}{2log(n)}$, 
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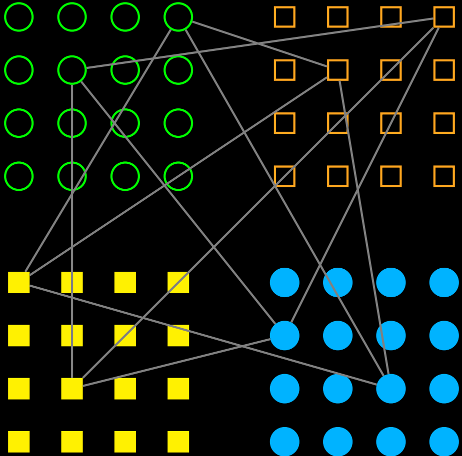
- Efficiency $\approx \frac{1}{log(n)log_2(M+1)}$.

Example with $n = 790$: 

Our model: storing

- Example: $c = 4$ clusters made of $l = 16$ fanals each,
- $\underbrace{8}_{j_1 \text{ in } c_1}\underbrace{3}_{j_2 \text{ in } c_2}\underbrace{2}_{j_3 \text{ in } c_3}\underbrace{9}_{j_4 \text{ in } c_4}$,



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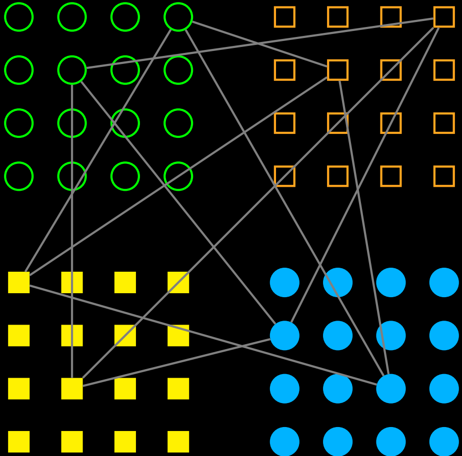

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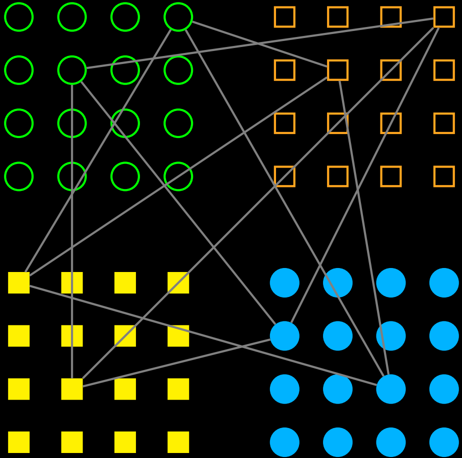

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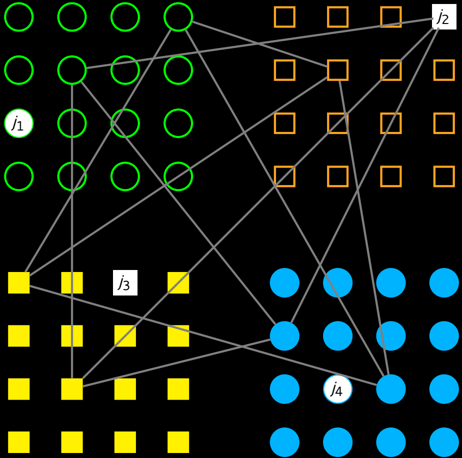

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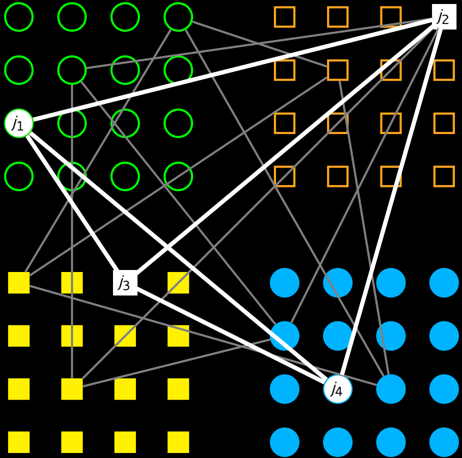

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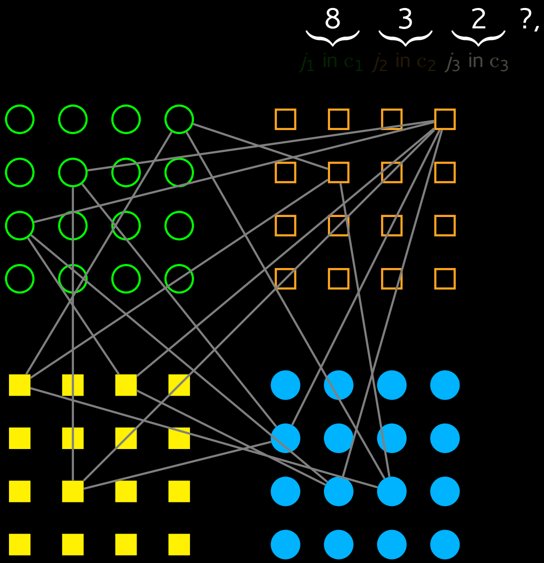

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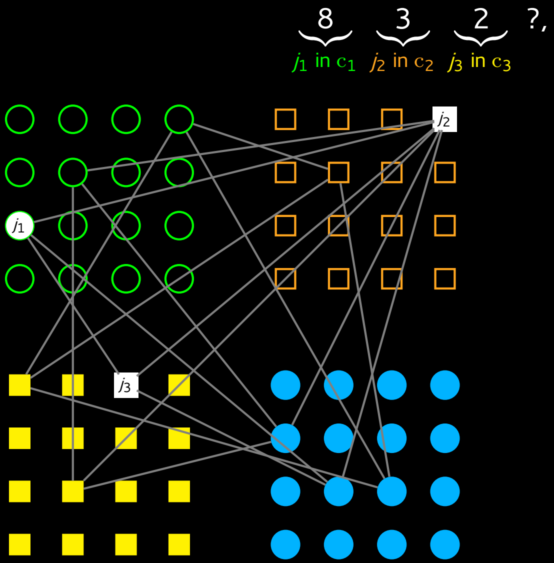

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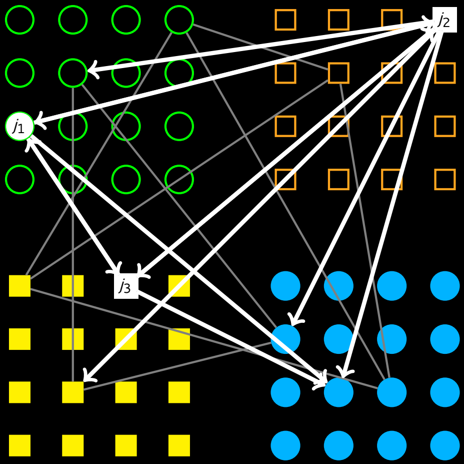

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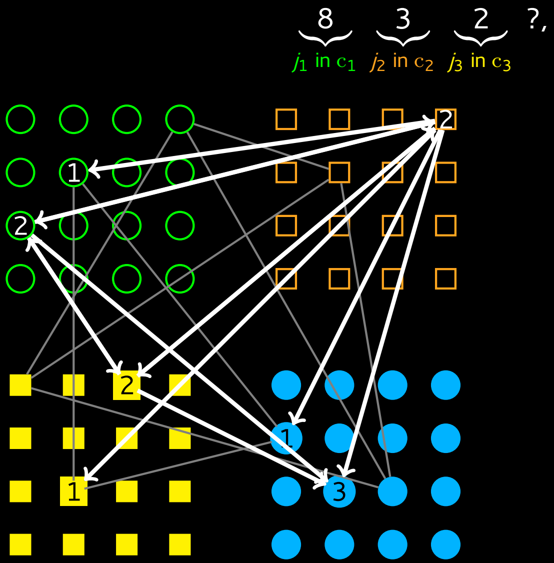

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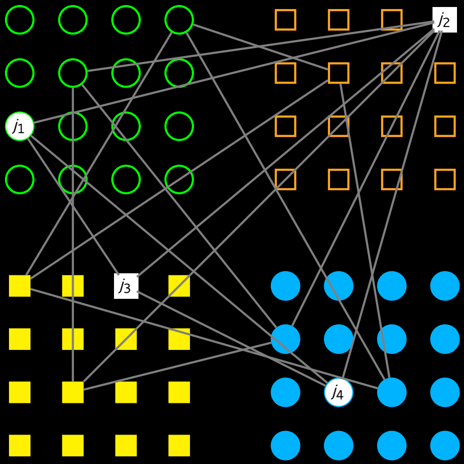

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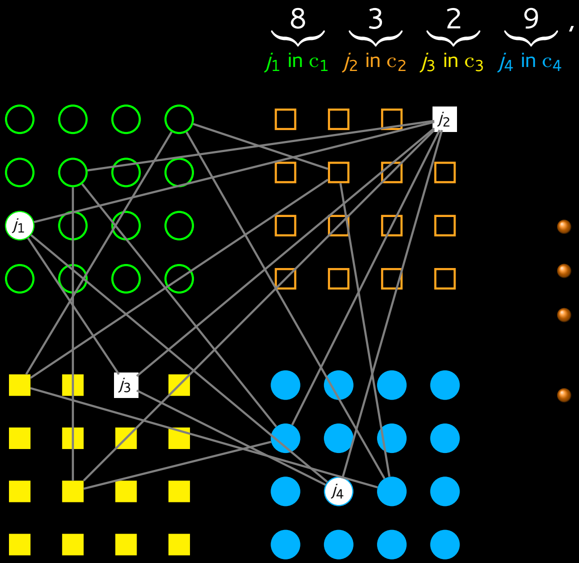

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Density

A binary model of long term memory

- Density d is the ratio of the number of used connections to the total number of possible ones,
- If messages are i.i.d.: $d \approx 1 - \left(1 - \frac{1}{l^2}\right)^M$.

Curves

Remarks

- $d = 1$: no more distinction between stored and not stored messages,
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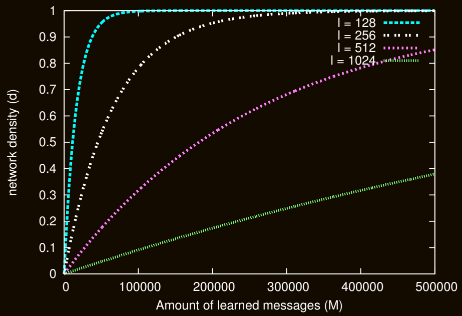

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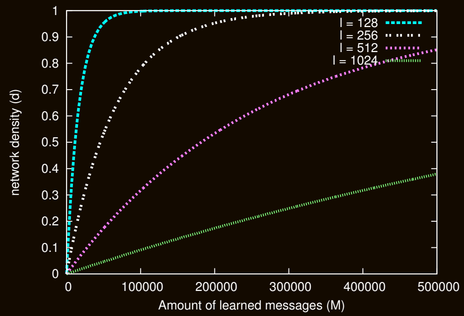

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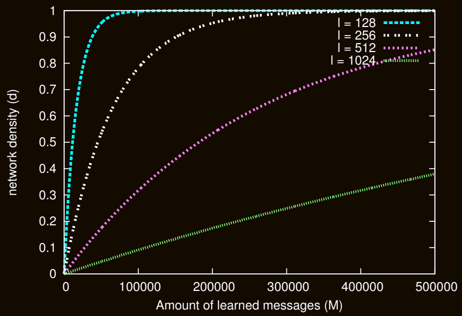

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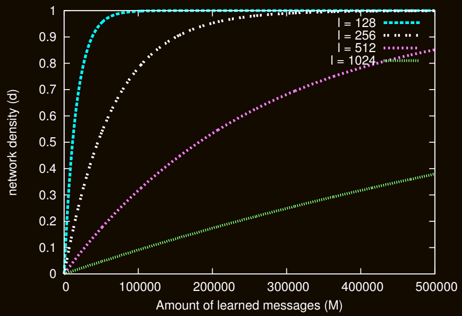

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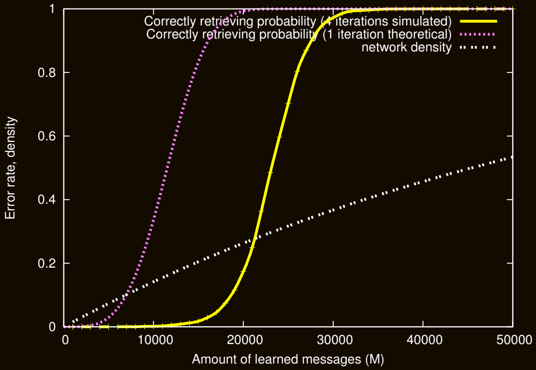
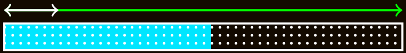
As an associative memory



$c = 8$ clusters of $l = 256$ fanals each ($\sim$ messages of 64 bits),
Error probability when retrieving messages half erased.

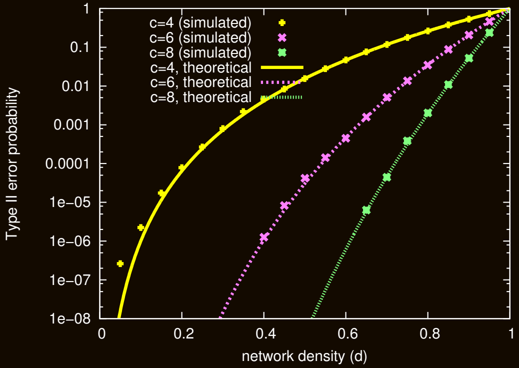
Hopfield network ($n = 790$)

Our network



Performance

Set implementation



Second kind error rate for various sizes of clusters c and for $l = 512$ fanals per cluster.

Hopfield network ($n = 740$)

Our network

Plan

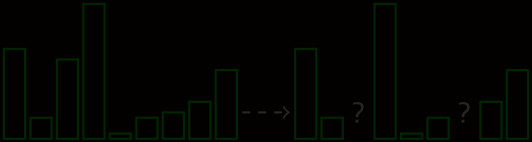
1. Biology and error correction codes
   - Starting point
   - From the neocortex to recurrent graphs : our three hypotheses
   - A few words about error correcting codes
2. A new architecture of associative memory
   - Storing
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Learning sequences
4. Current and future work
   - Storing hierarchical messages
   - Combining associative memories and classifiers
   - Towards new computation models based on information

Blurred messages

Limitation

Partial messages must contain perfect information.

Noise model



Soft decoding

Blurred messages

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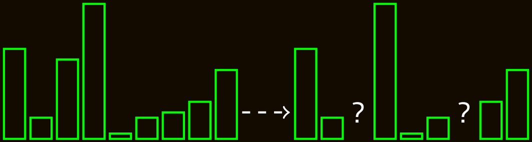

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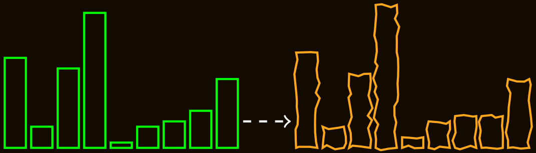

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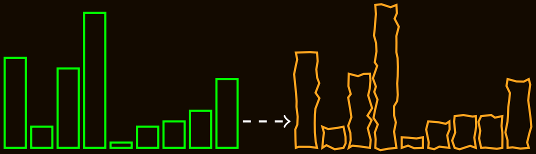

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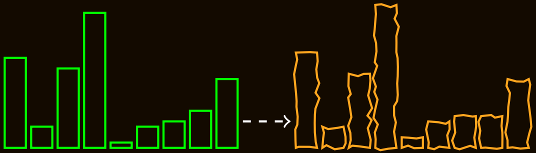

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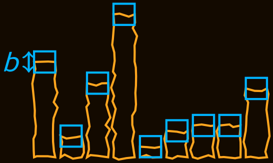

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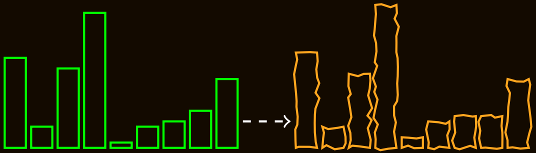

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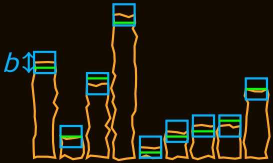
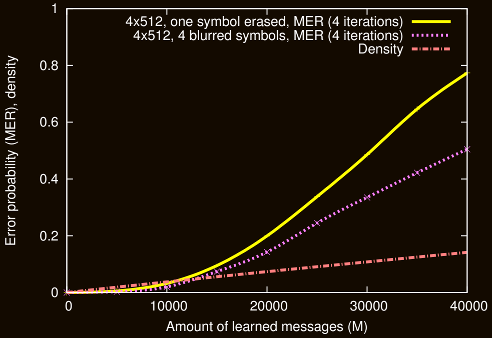


Performance

Simulations



Comparison of performance when messages are partially erased and when they are blurred ($b = 5$).

Correlated messages

Limitation

With correlations grows the number of Type II errors.

Fighting correlation by adding random redundancy

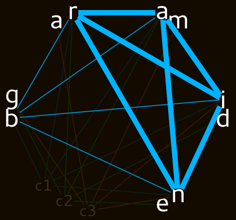
Correlated messages

Limitation

With correlations grows the number of Type II errors.

Fighting correlation by adding random redundancy

brain



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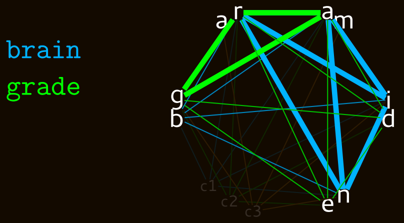

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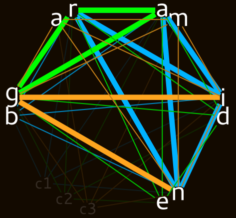
gamin



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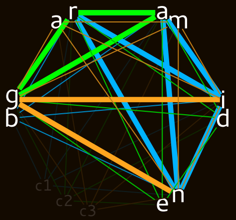
grade
gamin
grain



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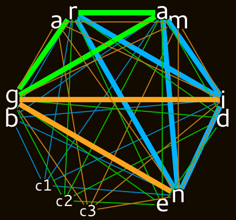
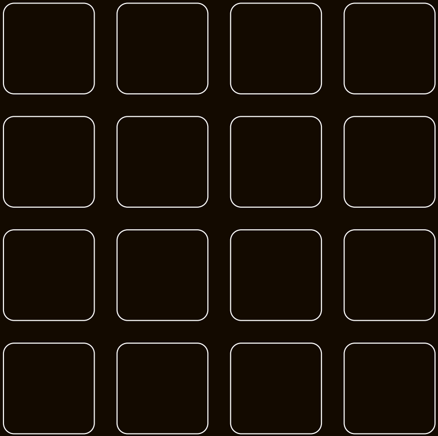
brain +c1
grade +c2
gamin +c3
grain +c?



Global winner-take-all

Illustration



Idea

- After global message passing...
- After local maximum selections...
- Global maximum selection.

Interests

- Diversity $\propto c^2$,
- Stored messages length may vary.

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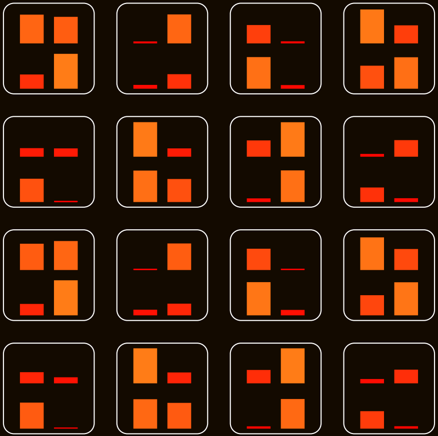

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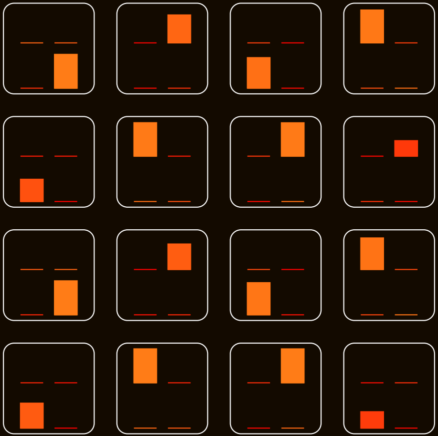

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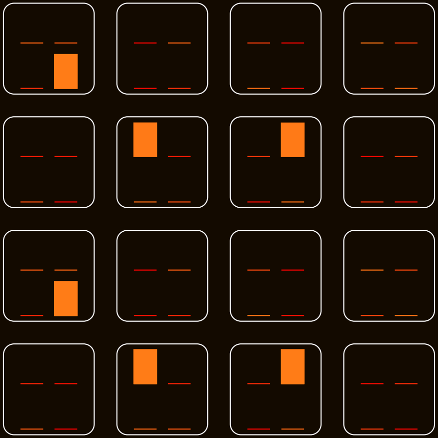

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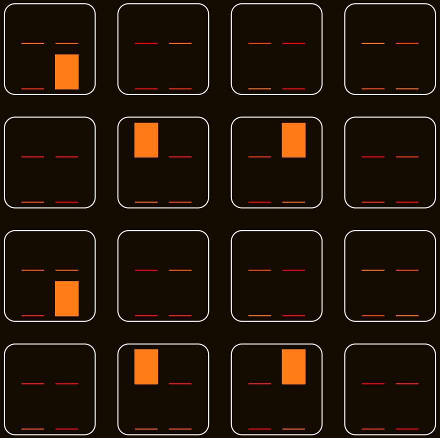

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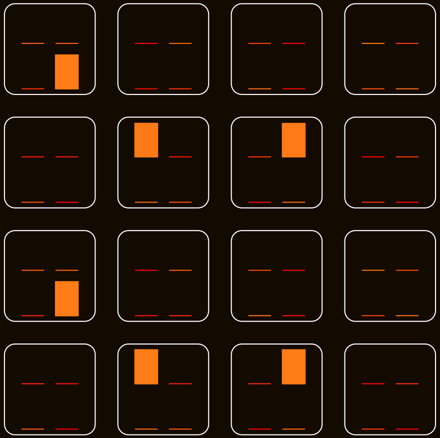

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Tournament chains and unidirectional connections

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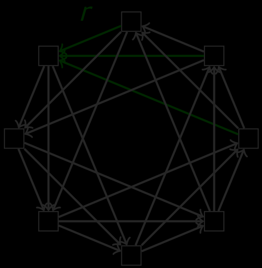
Bidirectional connections and full inter-connectivity.



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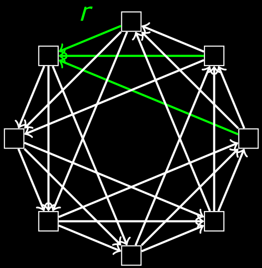

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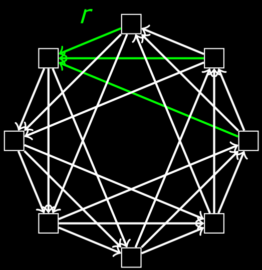
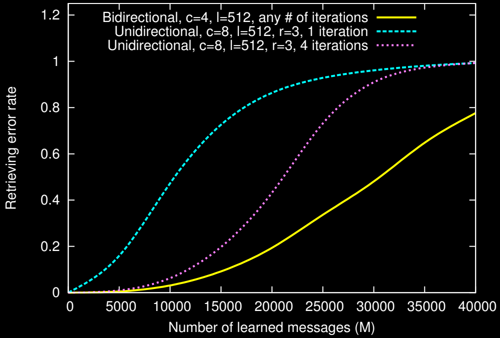
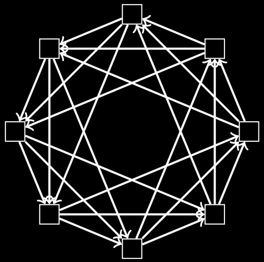




Learning arbitrarily long sequences



Performance

- $c = 50$ clusters,
- $l = 256$ fanals/cluster,
- $L = 1000$ symbols in messages,
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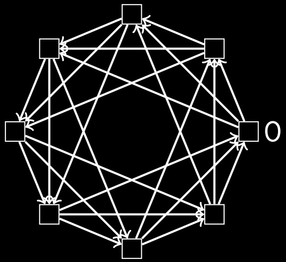
Learning arbitrarily long sequences



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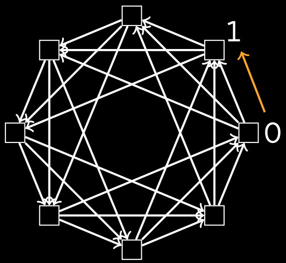

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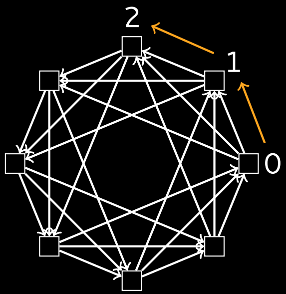

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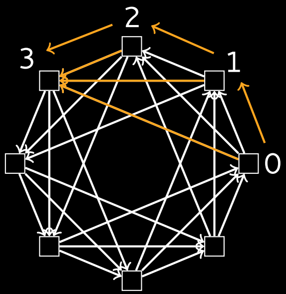

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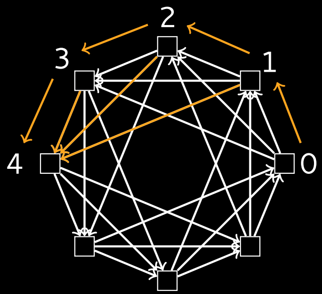

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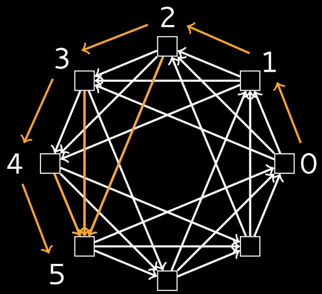

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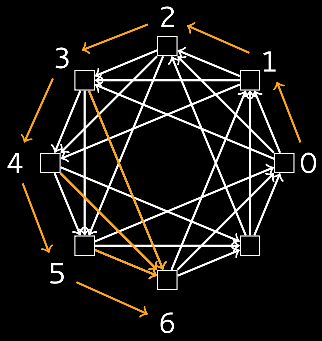

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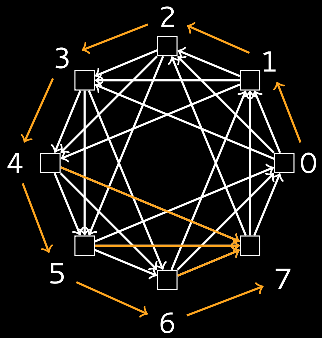

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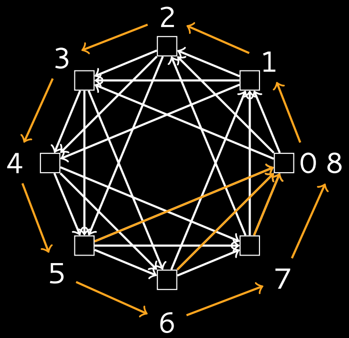

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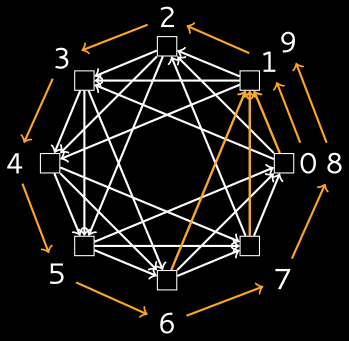

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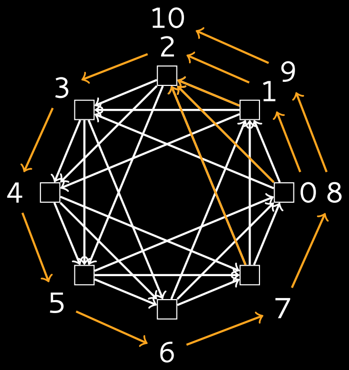

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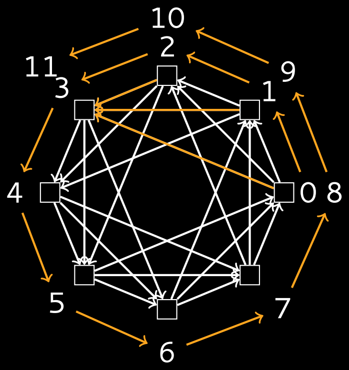

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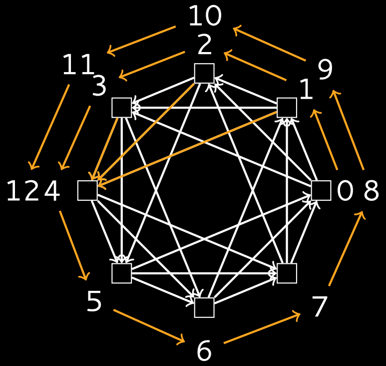

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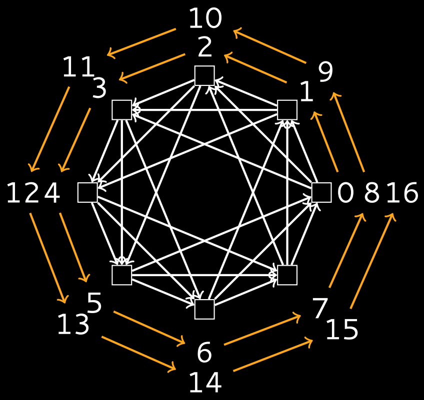

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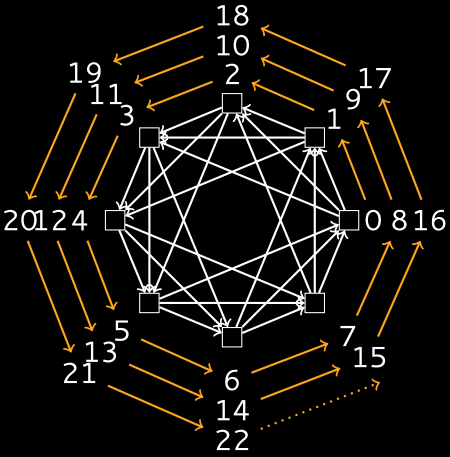

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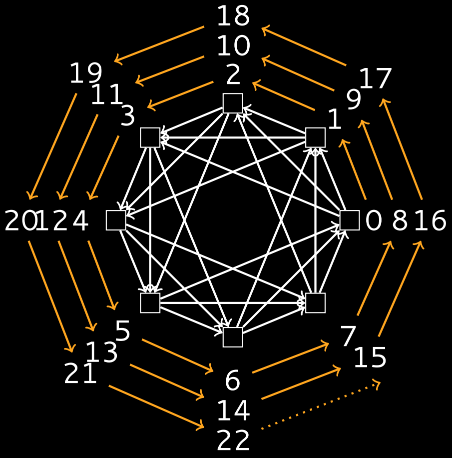

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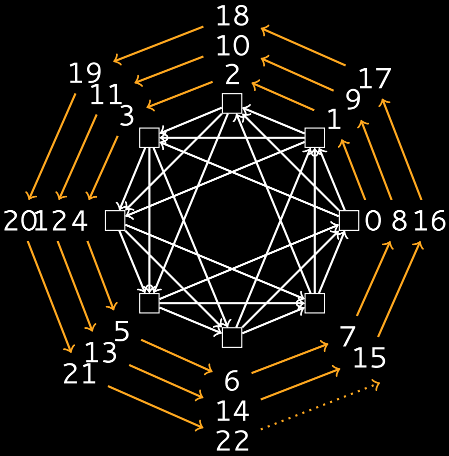

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Plan

1. Biology and error correction codes
   - Starting point
   - From the neocortex to recurrent graphs : our three hypotheses
   - A few words about error correcting codes
2. A new architecture of associative memory
   - Storing
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Learning sequences
4. Current and future work
   - Storing hierarchical messages
   - Combining associative memories and classifiers
   - Towards new computation models based on information

Storing hierarchical messages

Question

How to store pieces of hierarchical information (e.g. sentences of words of letters) into associative memories?

Idea

- Provide networks with a third dimension,
- Connect layers using subsampling,
- Use time in decoding.

Possibility to store hierarchical messages with unchanged efficiency.

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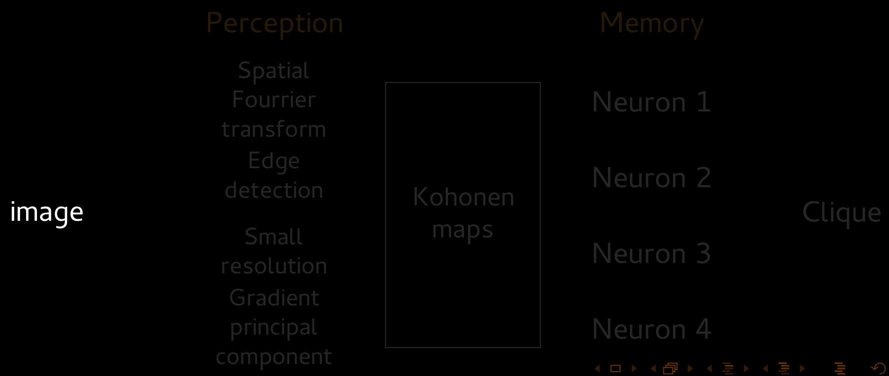

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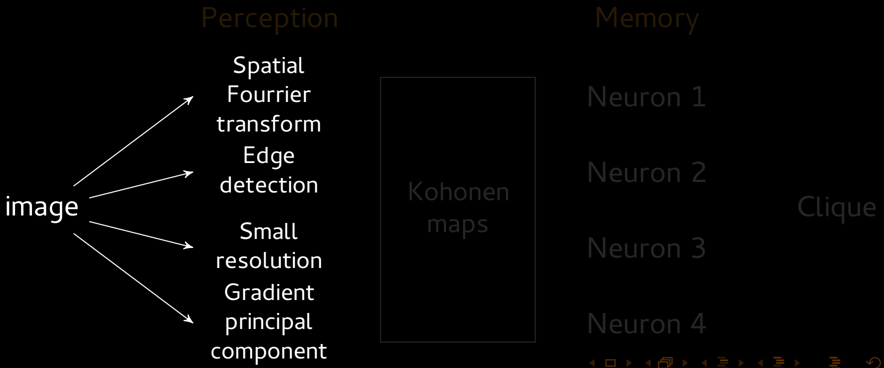

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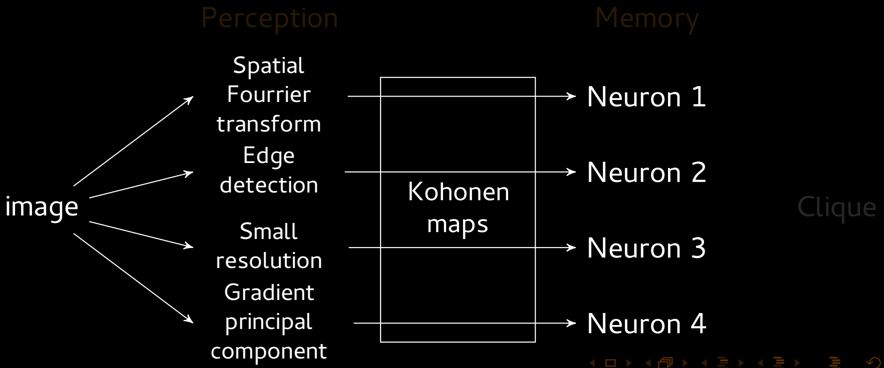

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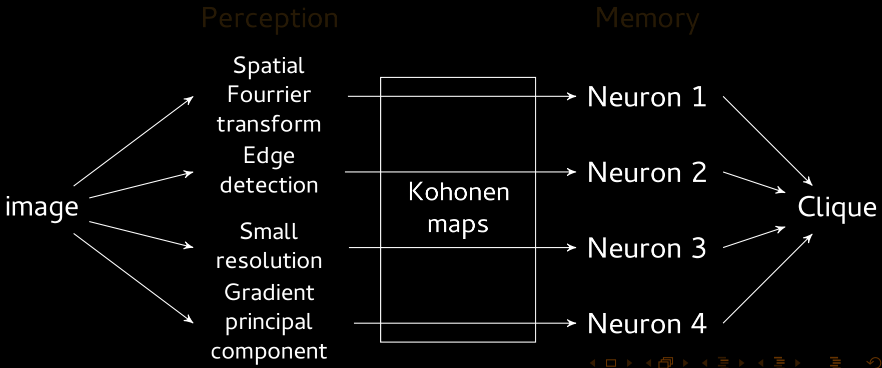

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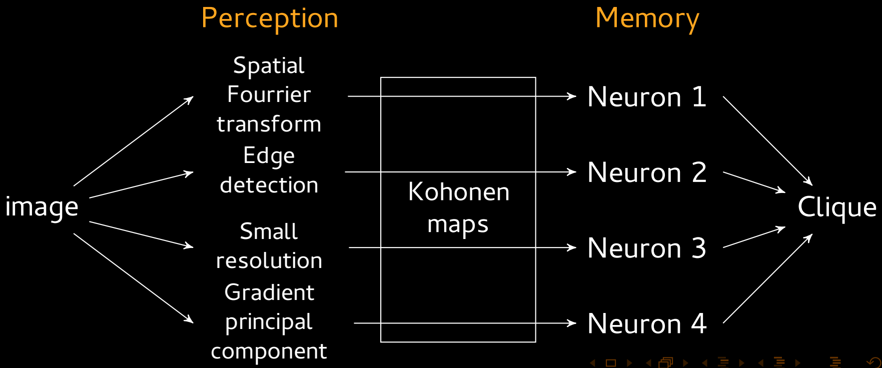

Towards new computation models based on information

Idea

- Finite input/output problem $\equiv$ finite set of couples $(input, output)$,
- Store them into an associative memory to obtain a solution.

Perspectives

- Extend to nonfinite input/output problems,
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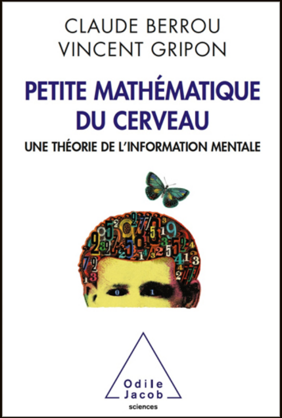
Questions

I am at your disposal if you have any question.

A bit of reading



To learn more

Visit:
http://www.vincent-gripon.com/?p1=100

Acknowledgements

Collaborators:

- C. Berrou,
- A. Abudib, X. Jiang,
- G. Coppin, D. Pastor, E. Sedgh Gooya.