Networks of neural cliques

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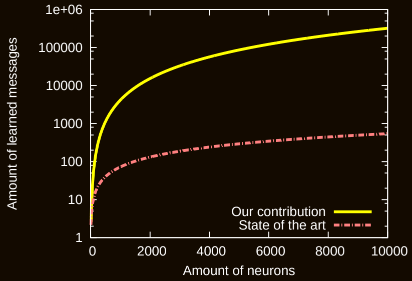
McGill University

2011, Nov. 11th

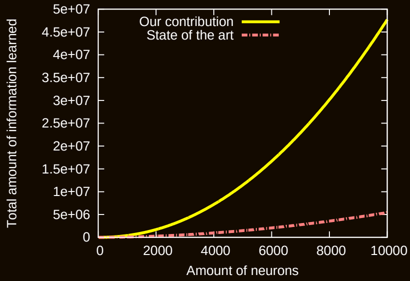
In a word...

Learning messages in recurrent neural networks

Learning diversity



Learning capacity



Our contribution

Hopfield neural networks

In a word. . .

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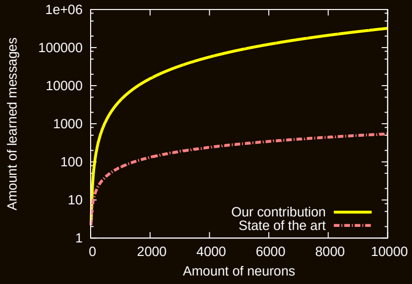

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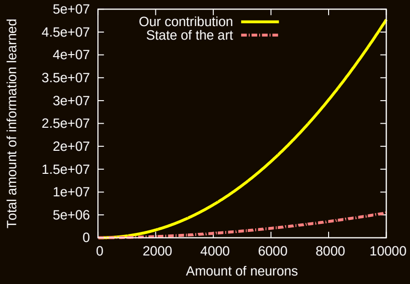

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Sparsity

Error correcting code

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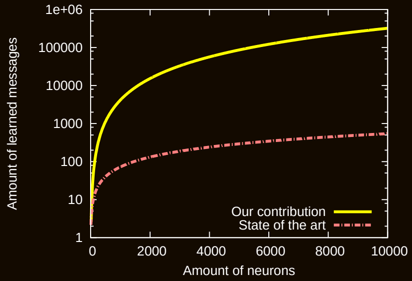

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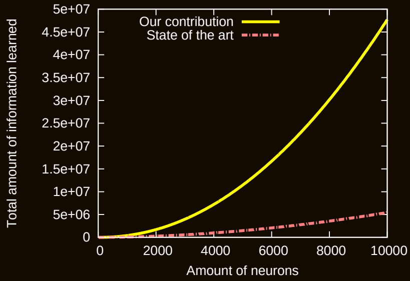


Our contribution

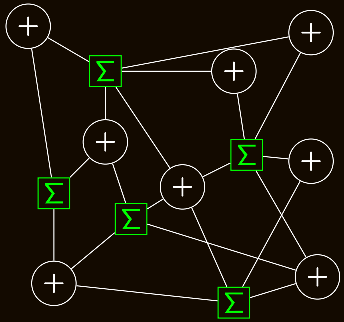
Sparsity
Error correcting code

Biologic plausibility

Hopfield neural networks

Starting idea

LDPC decoder



Neocortical “decoder”

decoding	=	remembering
nodes $\boxed{\Sigma}$	=	neurons
parity	=	?
?	=	learning

Outline

1. Associative memories and error correcting codes
   - Associative memory
   - Error correcting codes
   - Code of cliques
2. Sparse networks, principles and performance
   - Learning
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Sparse messages
   - Learning sequences
4. Conclusion

Plan

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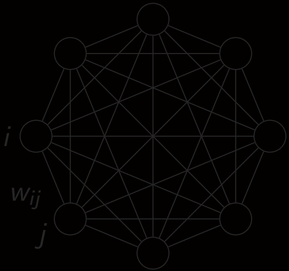
Associative memories

Two operations:

- Learning messages,
- Retrieving previously learned messages from part of their content.

State of the art: the Hopfield network



- Learning: M binary messages $\mathbf{d}^m$:

$$w_{ij} = \sum_{m=1, i \neq j}^{M} d_i^m d_j^m,$$

- Retrieving: iterates

$$\forall i, v_i \leftarrow \mathrm{sgn}\left(\sum_{j \neq i} v_j w_{ij}\right).$$

Associative memories and the Hopfield network

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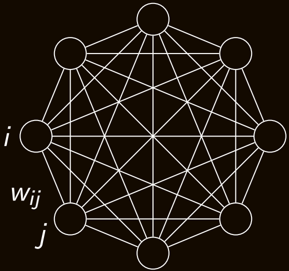

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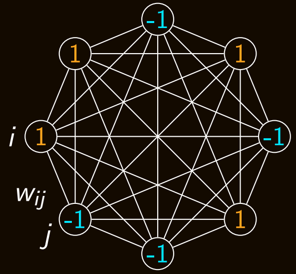

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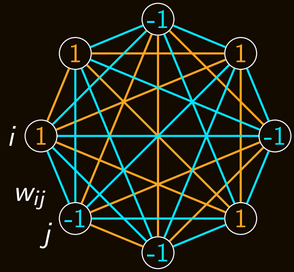

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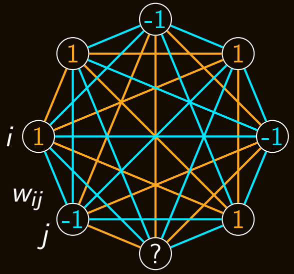

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Performance and bounds

Hopfield networks (n neurons $\longleftrightarrow$)

- Diversity : $M = \frac{n}{2log(n)}$, $\leftrightarrow$
- Capacity : $\frac{n^2}{2log(n)}$, $=$
- Total amount of required memory: $\binom{n}{2} log_2(M+1)$,
- $\Rightarrow$ Efficiency $\approx \frac{1}{log(n)log_2(M+1)}$.
- Sensitive connections, length of messages = size of the network, messages and their inverse are learned at the same time. . .

Example with $n = 790$: 

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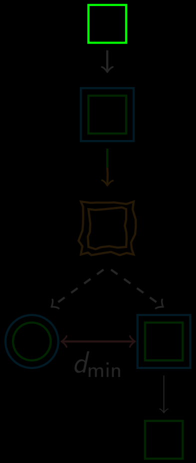
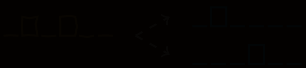


Example: the thrifty code

- Code containing only binary words with a single "1":

- Drawback: $d_{\min} = 2$:



- But easy to decode and minimise the energy:

- These codes can be associated like the distributed codes. . .

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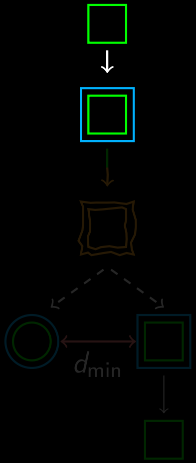

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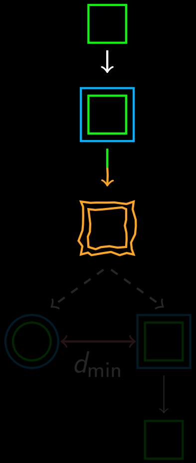

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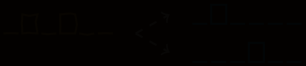

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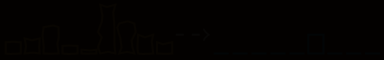

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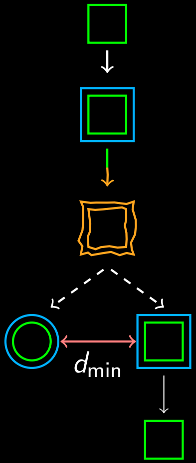

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winner-take-all

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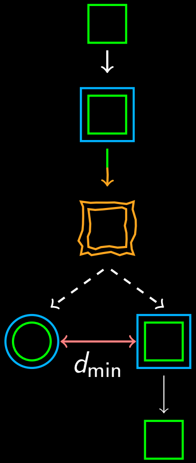

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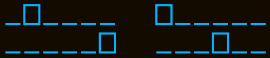

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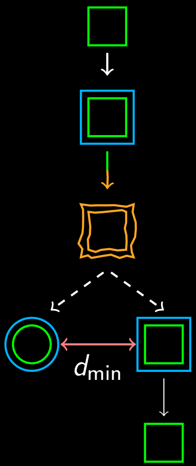

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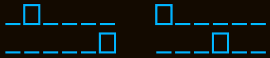

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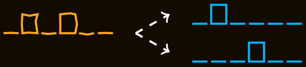

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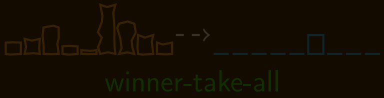

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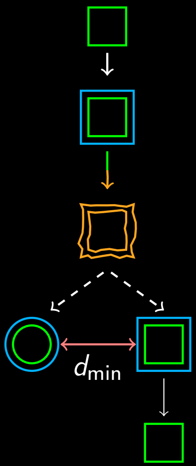

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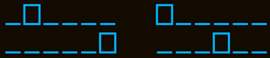

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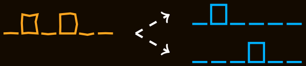

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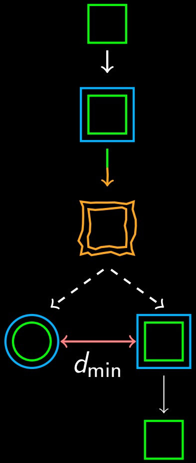

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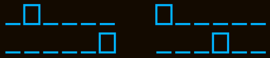

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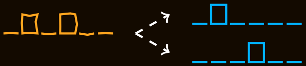

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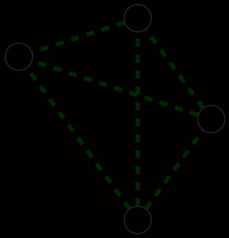
Example: codewords = 4 nodes cliques



2 distinct nodes
$\Rightarrow d_{\min} = 6$ edges

Clique

Set of nodes that are all connected one to another.

Codes of cliques of size $c \ll n$

- $d_{\min} = 2(c-1) \approx 2c$,
- $\Rightarrow F = r d_{\min} \approx 2$,
- Cliques are codewords of a very interesting error correcting code.

Codes made of cliques of constant size

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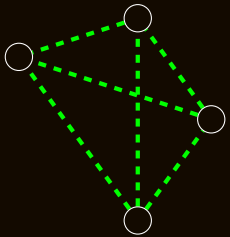

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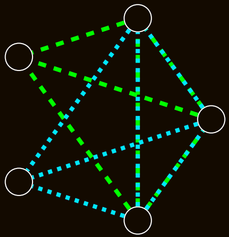

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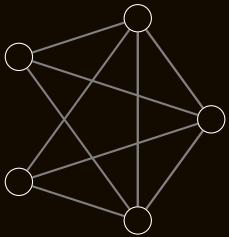

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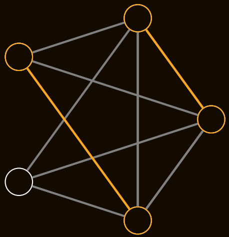

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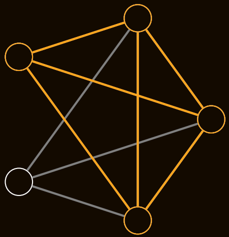

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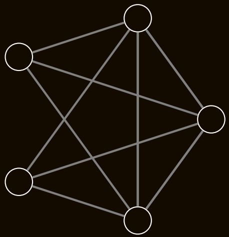

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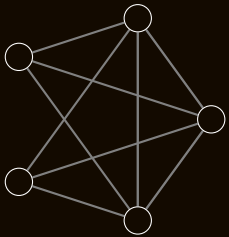

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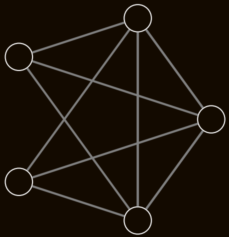

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Our model: learning

- Example: $c = 4$ clusters made of $l = 16$ neurons each,
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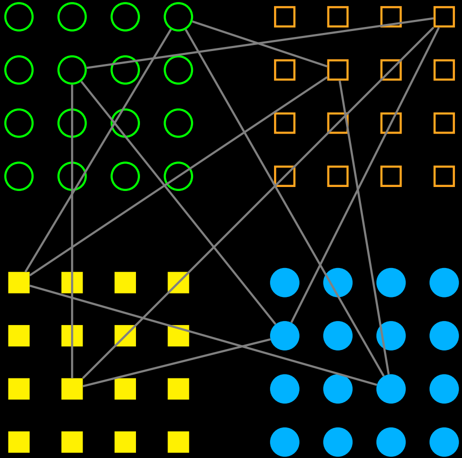

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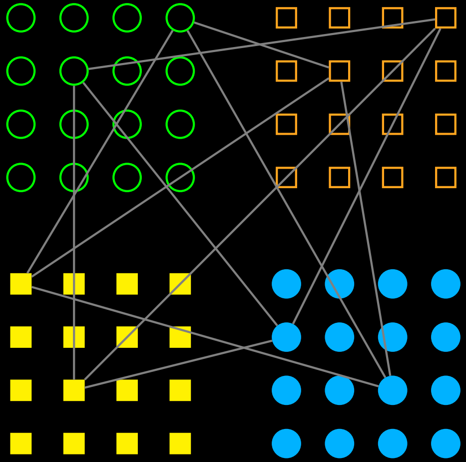

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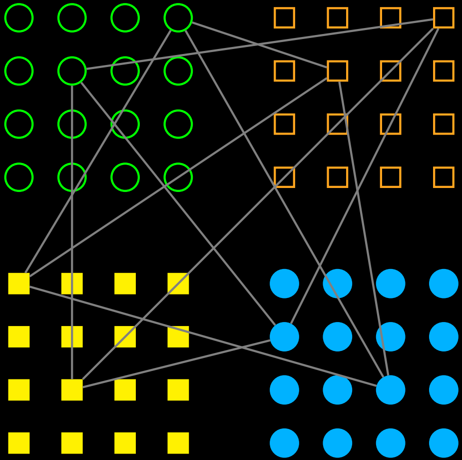

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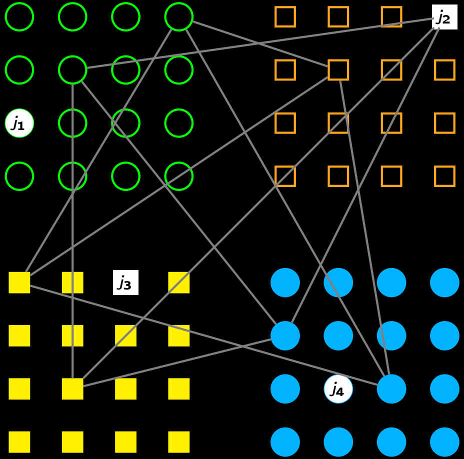

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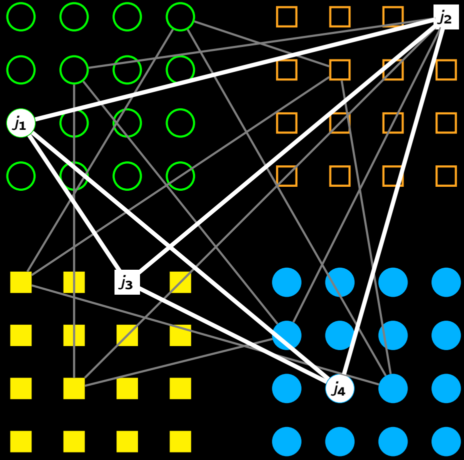

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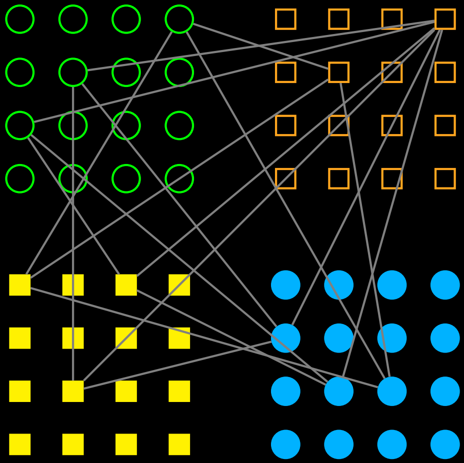

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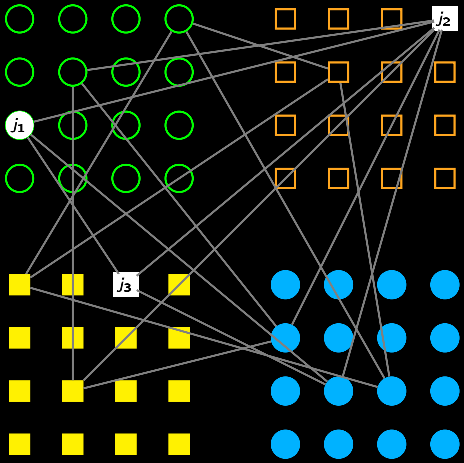

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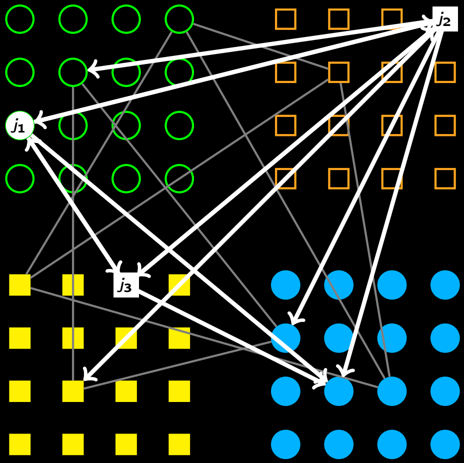

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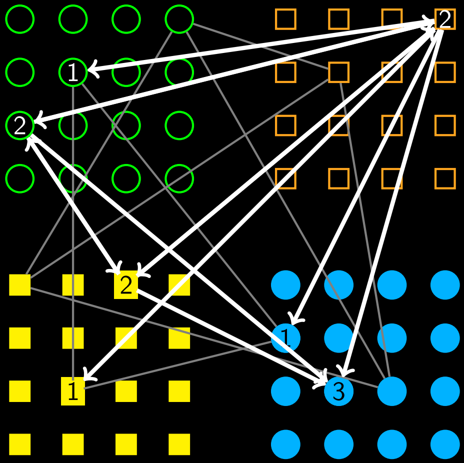

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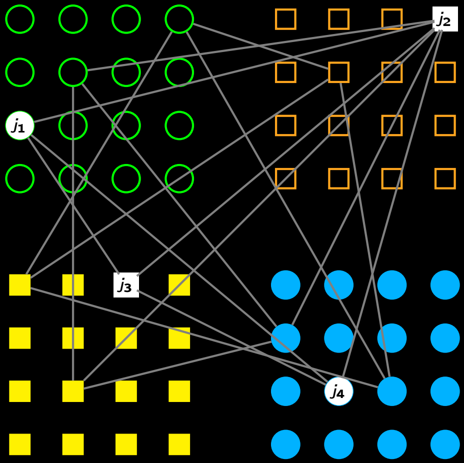

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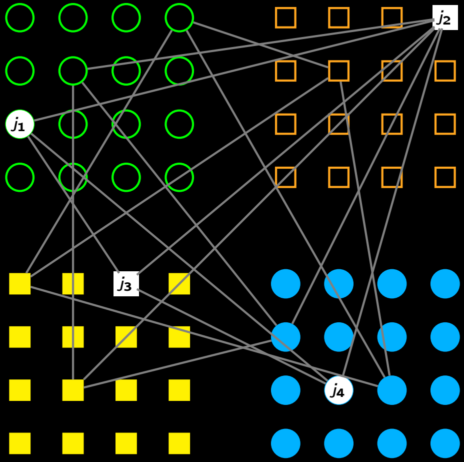

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Density

Introducing a new parameter

- Density d is the ratio of the number of used connections to the total number of possible ones,
- If messages are i.i.d.: $d \approx 1 - \left(1 - \frac{1}{l^2}\right)^M$.

Curves

Remarks

- $d = 1$: no more distinction between learned and not learned messages,
- $d = f(l, M)$, not depending on c,
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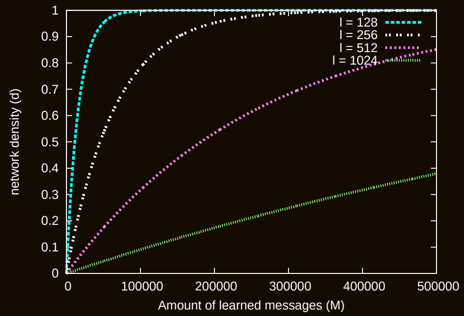

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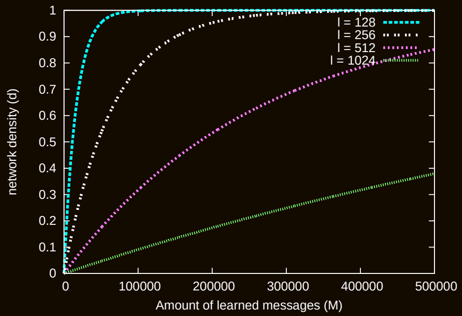

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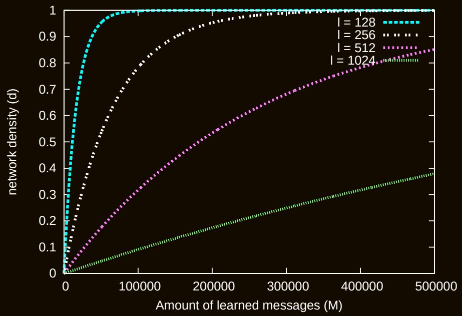

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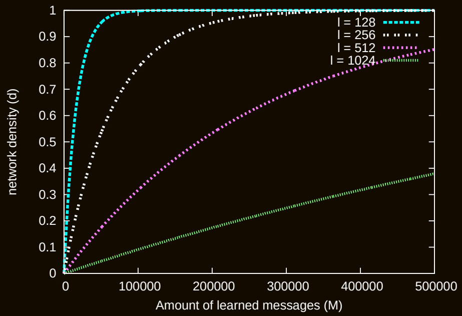

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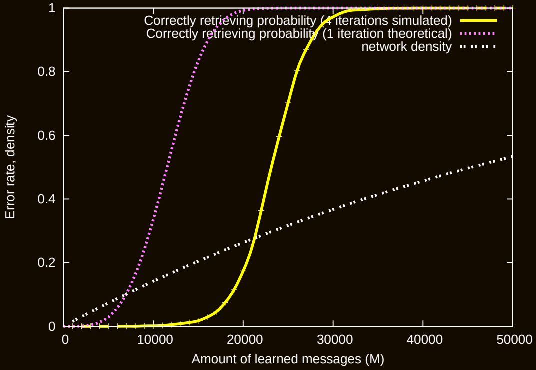
Performance (1/3)

As an associative memory

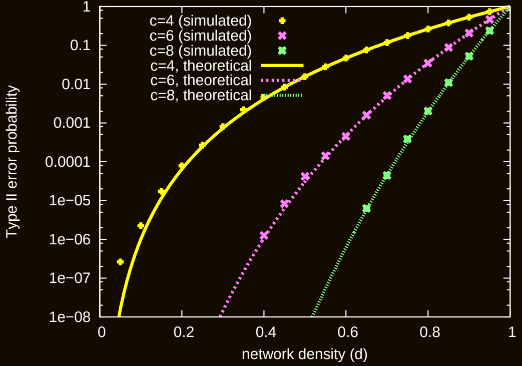


$c = 8$ clusters of $l = 256$ neurons each ($\sim$ messages of 64 bits),
Error probability when retrieving messages half erased.

Hopfield network ($n = 790$)

Our network

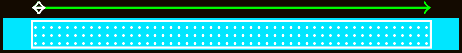
Set implementation



Second kind error rate for various sizes of clusters c and for $l = 512$ neurons per cluster.

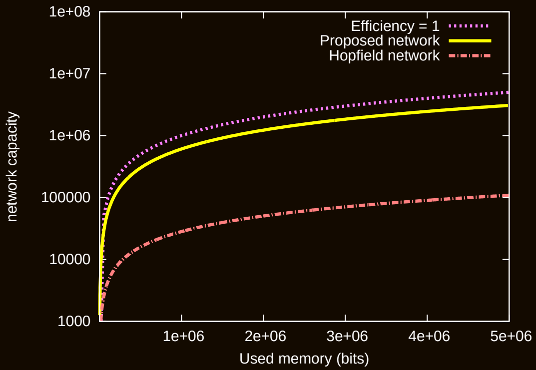
Hopfield network ($n = 740$)

Our network



Comparison of capacities of our network and of the Hopfield one

Performance (3/3)



Comparison of the capacities of the Hopfield network with ours (as associative memories) and for the same amount of memory used.

A word about plausibility

Analogies

Our network		Neuroscience litterature
Cliques of neurons	$\leftrightarrow$	Neural cliques
Local decoding	$\leftrightarrow$	Winner-take-all
Clusters	$\leftrightarrow$	Neocortical columns
Thrifty code	$\leftrightarrow$	Specific neurons

Limitations

- Necessity to provide a perfect - yet incomplete - content,
- Messages must not be correlated,
- Clusters must be large and few,
- Constant messages length,
- Systematic use of all clusters,
- Bidirectional connections and full inter-connectivity.

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Plan

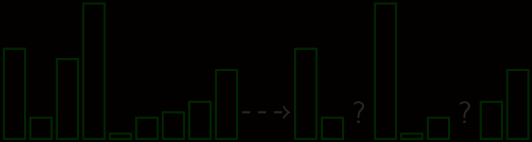
1. Associative memories and error correcting codes
   - Associative memory
   - Error correcting codes
   - Code of cliques
2. Sparse networks, principles and performance
   - Learning
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Sparse messages
   - Learning sequences
4. Conclusion

Blurred messages

Limitation

Partial messages must contain perfect information.

Noise model



Soft decoding

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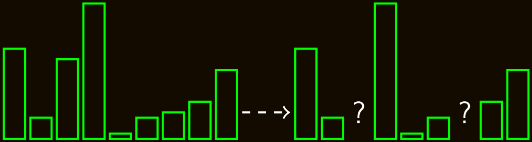

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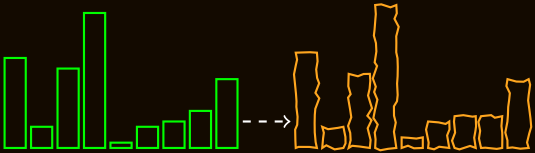

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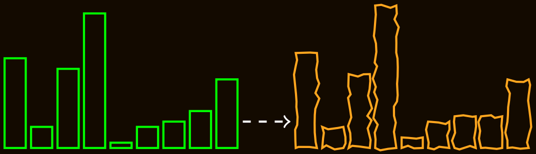

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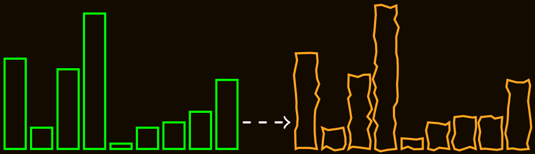

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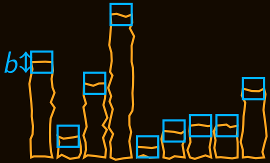

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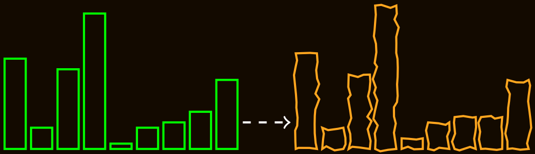

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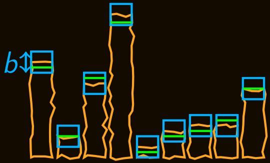
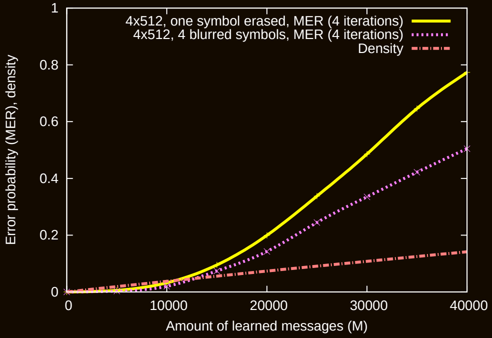


Performance

Simulations



Comparison of performance when messages are partially erased and when they are blurred ($b = 5$).

Why performance are better?

- Erasing: $\searrow$ competitive cliques ($\approx l$) $\nearrow$ probability ($\approx d^{c-1}$),
- Bruit : $\nearrow$ competitive cliques ($\approx b^c$) $\searrow$ probability ($\approx d^{\frac{c(c-1)}{2}}$).

Correlated messages

Limitation

With correlations grows the number of Type II errors.

Fighting correlation by adding random redundancy

- There are two effects of correlation:
  - An inescapable effect: *brain* and *train* are learned $\rightarrow$ $*$*rain* ?
  - Another effect coming from our network:

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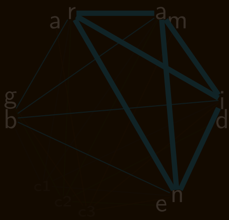

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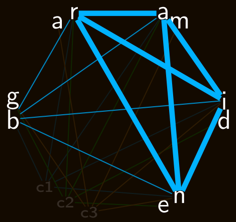

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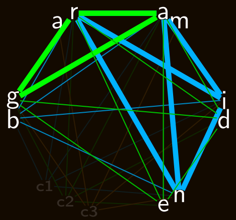

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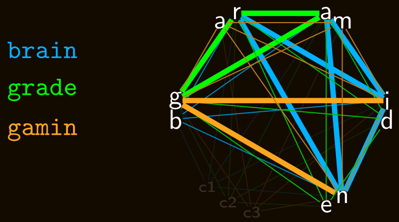

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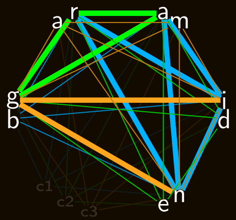
gamin
grain



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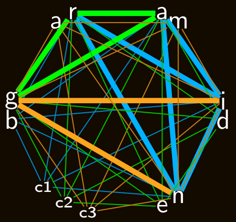
brain +c1

grade +c2

gamin +c3

grain +c?



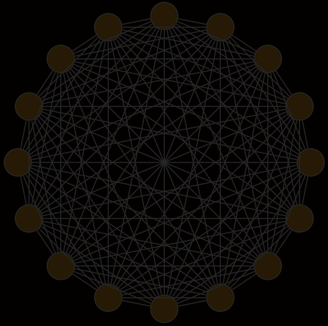
Towards a fourth level of sparsity

Limitations

- Clusters must be large and few,
- Learned messages are all of the same length.

Illustration

0000101000001011



Idea

1. Shorter messages,
2. Clusters and thrifty codes,
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4. Sparse messages.

Solution

- Global winner-take-all.

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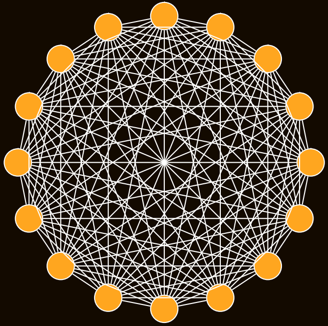

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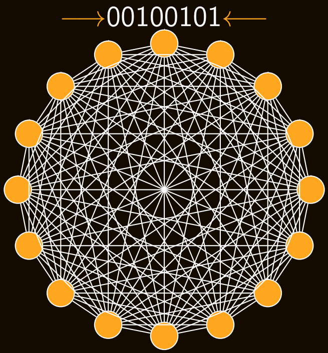

Idea

1. Shorter messages,
2. Clusters and thrifty codes,
3. Sparse network,
4. Sparse messages.

Solution

- Global winner-take-all.

Towards a fourth level of sparsity

Limitations

- Clusters must be large and few,
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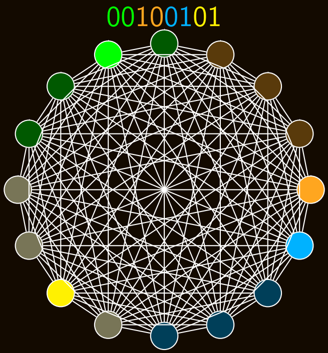

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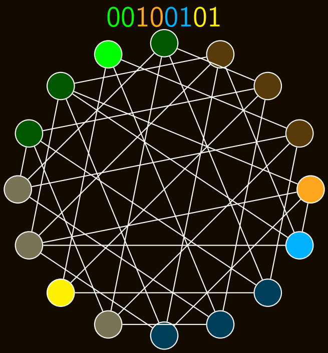

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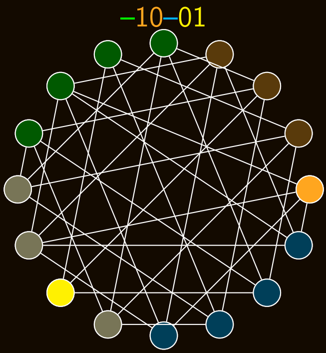

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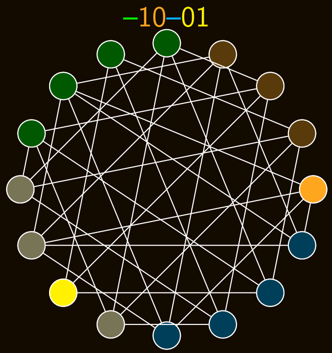

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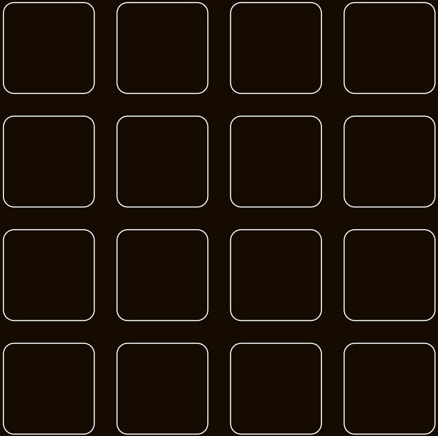

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Global winner-take-all

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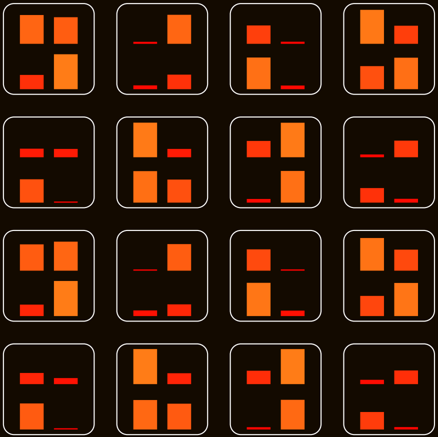

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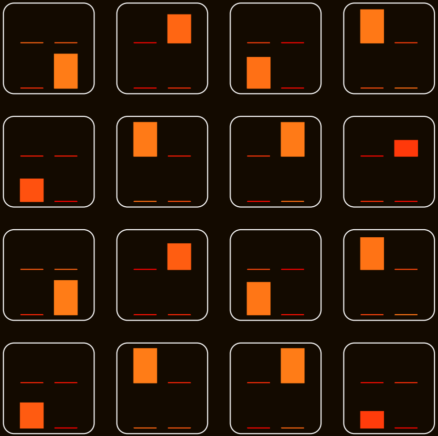

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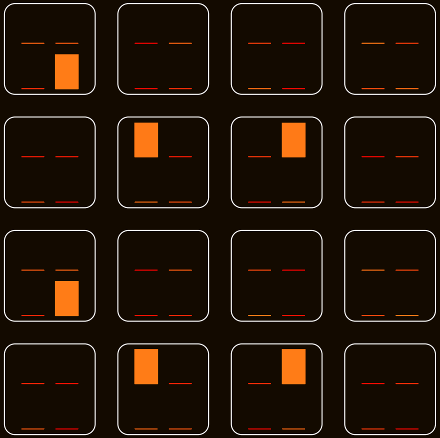

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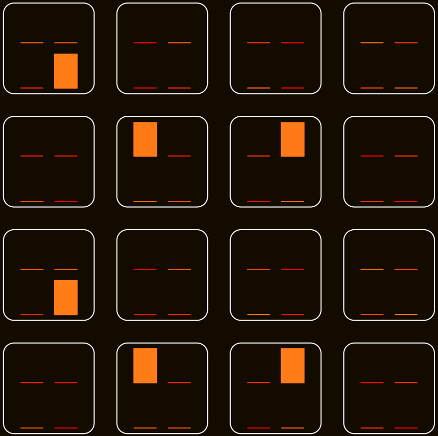

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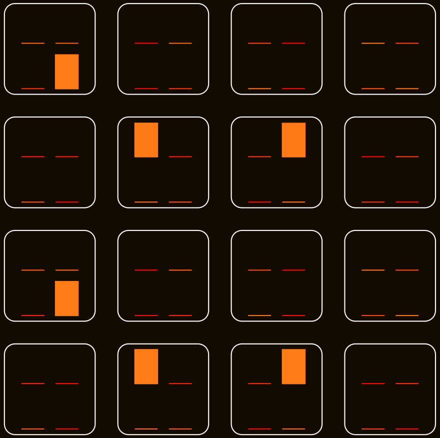

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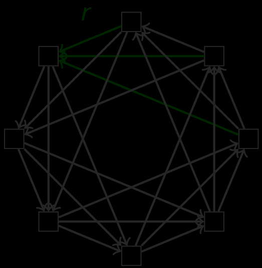

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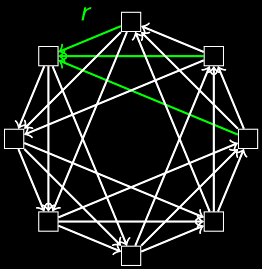

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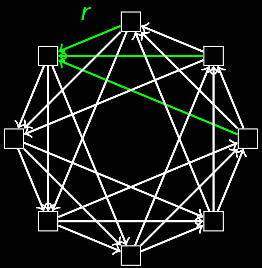

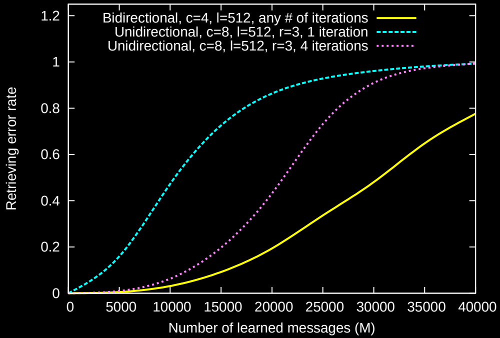

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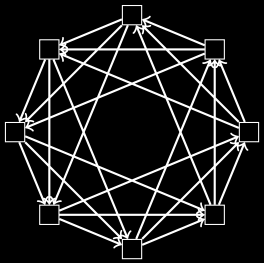

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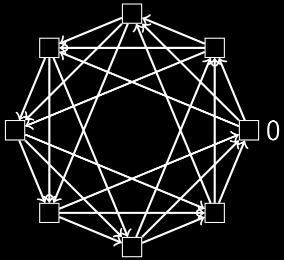

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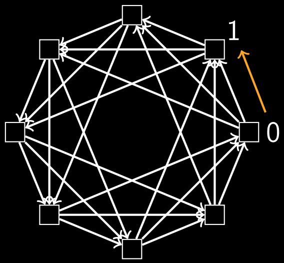

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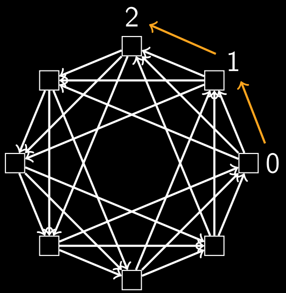

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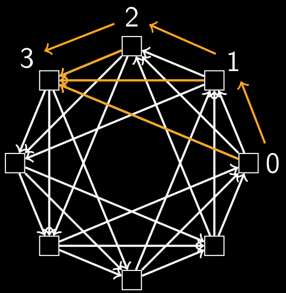

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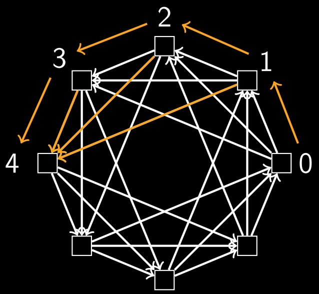

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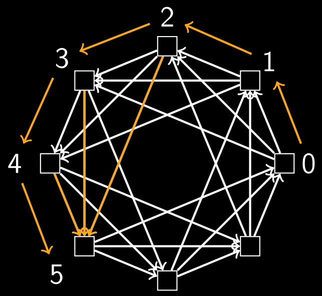

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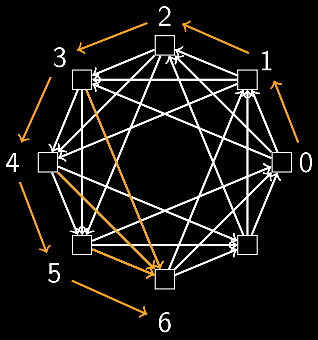

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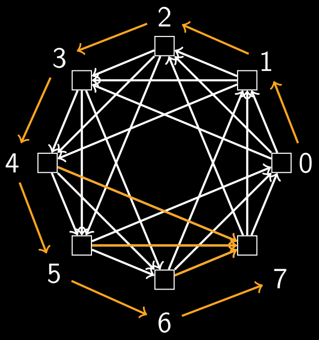

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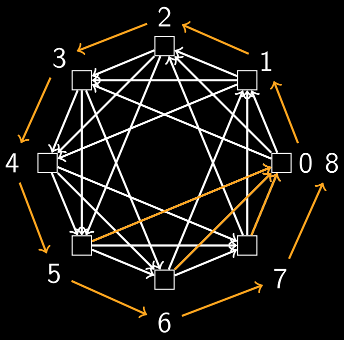

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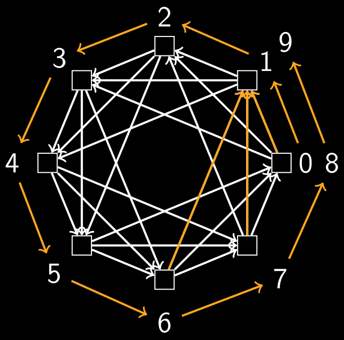

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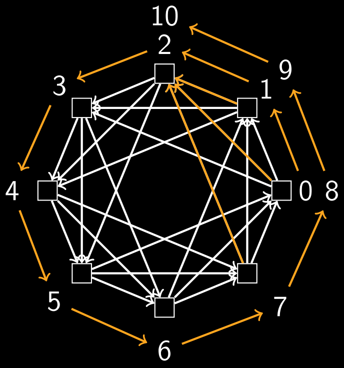

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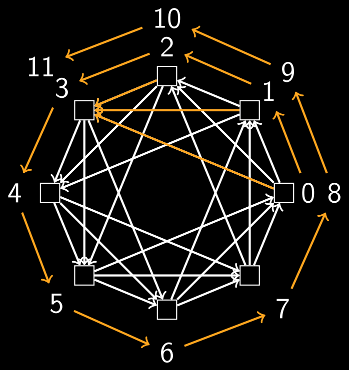

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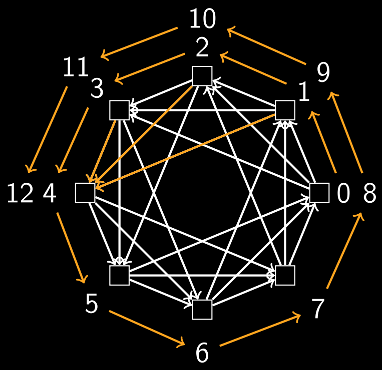

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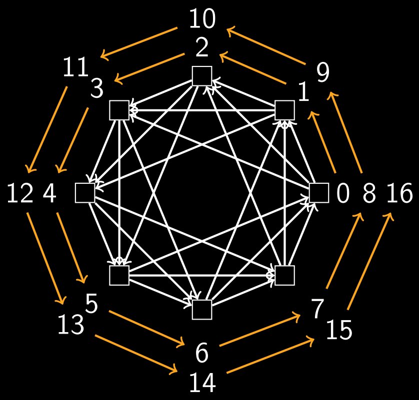

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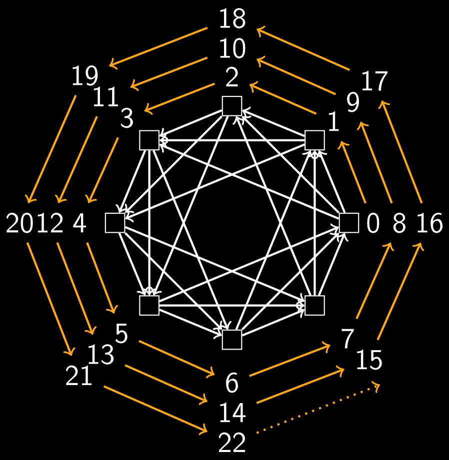

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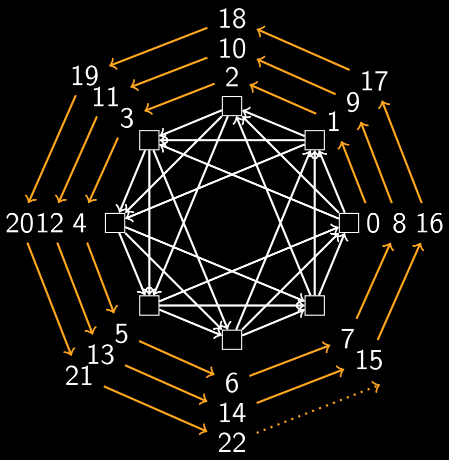

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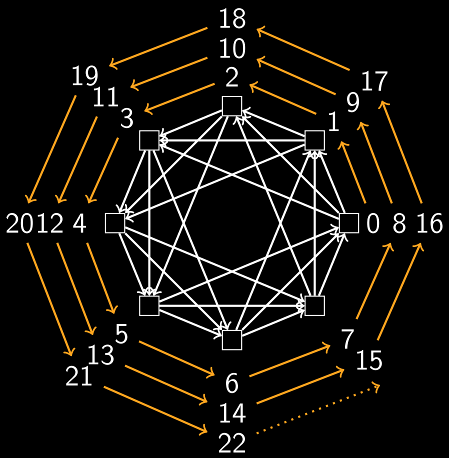

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Plan

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   - Associative memory
   - Error correcting codes
   - Code of cliques
2. Sparse networks, principles and performance
   - Learning
   - Retrieving
   - Performance
3. Developments
   - Blurred messages
   - Correlated sources
   - Sparse messages
   - Learning sequences
4. Conclusion

Conclusion

Approach

Designing an associative memory

neocortical architecture

sparsity

distributed codes

Results

- Nearly optimal capacities, substantial diversities,
- Massively parallel architecture,
- Analogies with neurobiological architecture and functioning,
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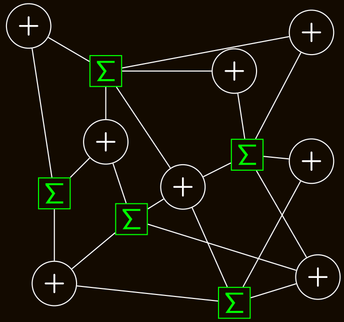
Thank you for your attention. I am at your disposal if you have any question.

LDPC decoder



Neocortical "decoder"